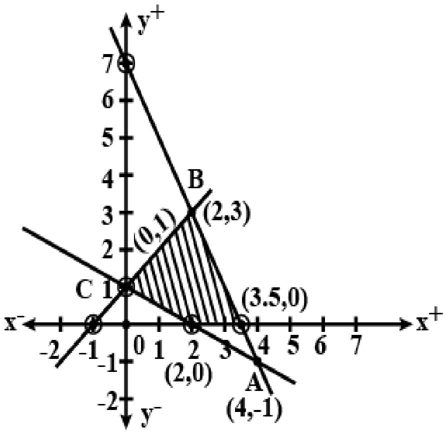
BCM SCHOOL LUDHIANA
CLASS XII
APPLICATION OF INTEGRALS
ANSWER KEY OF AOI

1

ar(△ABC)



$$= \int_0^2 y_{CB} dx + \int_4^2 y_{BA}. dx - \int_4^0 y_{AC}. dx$$

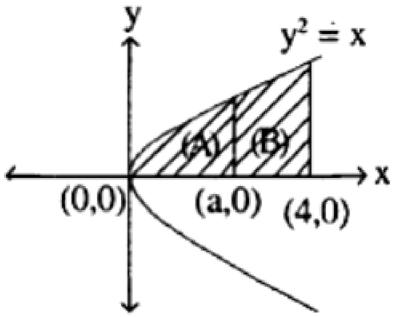
$$= \int_0^2 (x+1)dx + \int_0^2 (7-2x)dx - \int_4^0 (\frac{2-x}{2}) dx$$

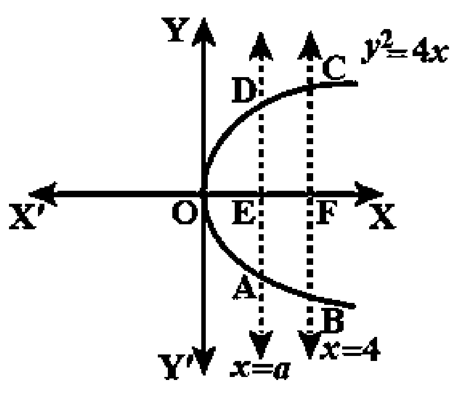
$$= [\frac{x^2}{2}+x]_0^2 + [7x - \frac{2x^2}{2}]_4^2 - \frac{1}{2}[2x - \frac{x^2}{2}]_4^0$$

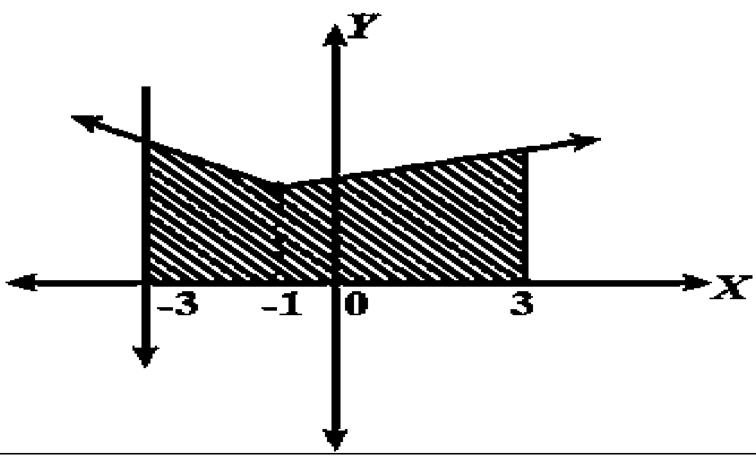
$$= [\frac{2^2}{2}+2] - [\frac{0^2}{2}+0] + \left[7(2) - \frac{2(2)^2}{2}\right] - \left[7(4) - \frac{2(4)^2}{2}\right] - \frac{1}{2}[2(0) - \frac{0^2}{2}] + \frac{1}{2}$$

$$= [2+2] - [0] + [14-4] - [28-16] - \frac{1}{2}[0] + \frac{1}{2}[8-8]$$

$= 2$ sq. units

2	As area between $y^2 = x$, $x = 4$ is divided into 2 parts by $x = a$,  so,(area under $y^2 = x$, $x = 0$ to $x = a$) = (area under $y^2 = x$, $x = a$, $x = 4$) $\Rightarrow$ area (A) = area (B) $\Rightarrow \int_0^a \sqrt{x}\, dx = \int_a^4 \sqrt{x}\, dx$ $\Rightarrow \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a = \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4$ $\Rightarrow \left.x^{\frac{3}{2}}\right]_0^a = \left.x^{\frac{3}{2}}\right]_a^4$ $(a = 2^{\frac{4}{3}})$

3	The line, x = a, divides the area bounded by the parabola and x = 4 into two equal parts. ∴ Area OAD = Area ABCD It can be observed that the given area is symmetrical about x-axis. ⇒ Area OED = Area EFCD Area OED = $\int_0^a y\,dx$ $= \int_0^a \sqrt{x}\,dx$ $= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$ $= \frac{2}{3}(a)^{\frac{3}{2}}$ .... (1) Area of EFCD = $\int_a^4 \sqrt{x}\,dx$ $= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4$ $= \frac{2}{3}[8 - a^{\frac{3}{2}}]$ .... (2) From (1) and (2), we obtain $\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3}[8 - (a)^{\frac{3}{2}}]$ $\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$ $\Rightarrow (a)^{\frac{3}{2}} = 4$ $\Rightarrow a = (4)^{\frac{2}{3}}$ Therefore, the value of a is $(4)^{\frac{2}{3}}$. 

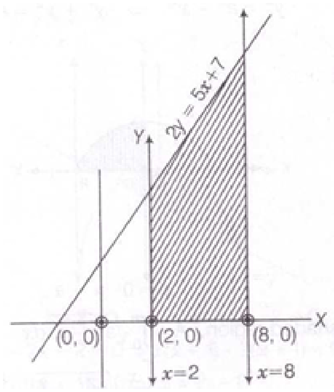
4	We have, $y = 1 + \|x+1\|, x = -3, x = 3, y = 0$ $\therefore y = \begin{cases} -x, & if \quad x < -1 \\ x+2, & if \quad x \geq -1 \end{cases}$ $\therefore$ Area of shaded region, $A = \int_{-3}^{-1} -x dx + \int_{-1}^{3} (x+2) dx$ $= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$ $= -\left[\frac{1}{2} - \frac{9}{2}\right] + \left[\frac{9}{2} + 6 - \frac{1}{2} + 2\right]$ $= -[-4] + [8+4]$ $= 12 + 4 = 16\ sq\ units.$ 

5

Using integration, find the area of the region bounded by the line $2y = 5x + 7$, $x - axis$ and the lines $x = 2$ and $x = 8$.

We have $2y = 5x + 7$

$$\Rightarrow y = \frac{5x}{2} + \frac{7}{2}$$



$\therefore$ Area of shaded region=

$$\frac{1}{2}\int_2^8 (5x+7)dx = \frac{1}{2}\left[5.\frac{x^2}{2} + 7x\right]_2^8$$

$$= \frac{1}{2}[5 \cdot 32 + 7 \cdot 8 - 10 - 14] = \frac{1}{2}[160 + 56 - 24]$$

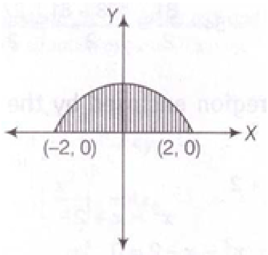
$$= \frac{192}{2} = 96\, sq\, units$$

6

Sketch the region $\{(x, 0): y = \sqrt{4 - x^2}\}$ and x-axis. Find the area of the region using integration.

Given region is $\{(x, 0): y = \sqrt{4 - x^2}\}$ and X-axis.

We have, $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$



$\therefore$ Area of shaded region, $A = \int_{-2}^{2} \sqrt{4 - x^2}\, dx = \int_{-2}^{2} \sqrt{2^2 - x^2}\, dx$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}.\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= \frac{2}{2}.0 + 2.\frac{\pi}{2} + \frac{2}{2}.0 - 2\sin^{-1}(-1) = 2.\frac{\pi}{2} + 2.\frac{\pi}{2}$$

$$= 2\pi\, sq\, units$$

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Find the area enclosed by the curve $x = 3\cos t,\ y = 2\ \sin t$.

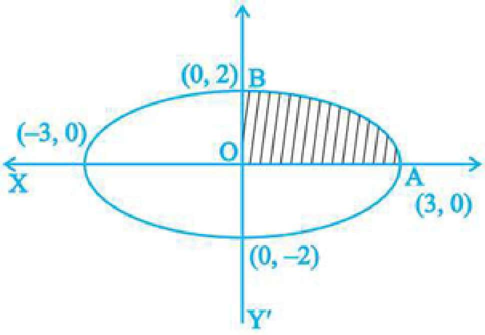

Fig. 8.5

Eliminating t as follows:

$x = 3\cos t,\ y = 2\ \sin t \Rightarrow \frac{x}{3} = \cos t, \frac{y}{2} = \sin\ t,$ we obtain $\frac{x^2}{9} + \frac{y^2}{4} = 1,$ which is the equation of an ellipse.

From Fig. 8.5, we get the required area $= 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\,dx$

$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 = 6\pi\, sq\, units.$

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Find the area of the region bounded by the curves $x = at^2\ and\ y = 2at$ between the ordinate coresponding to $t = 1\ and\ t = 2$.

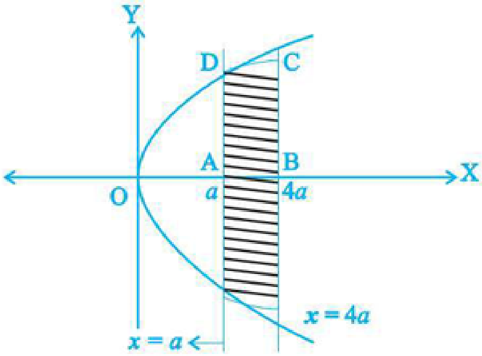

Fig. 8.7

Given that $x = at^2\ \ldots(i),\ y = 2at\ \ldots(ii) \Rightarrow t = \frac{y}{2a}$ putting the value of t in (i), we get

$y^2 = 4ax$

Putting $t = 1\ and\ t = 2$ in (i), we get $x = a,\ and\ x = 4a$

Required area = 2 area of ABCD = $2\int_a^{4a} y dx = 2\times 2\int_a^{4a}\sqrt{ax}\,dx$

$= 8\sqrt{a}\left|\frac{(x)^{\frac{3}{2}}}{3}\right|_a^{4a} = \frac{56}{3}a^2\ sq\, units.$

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9 **Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$.**

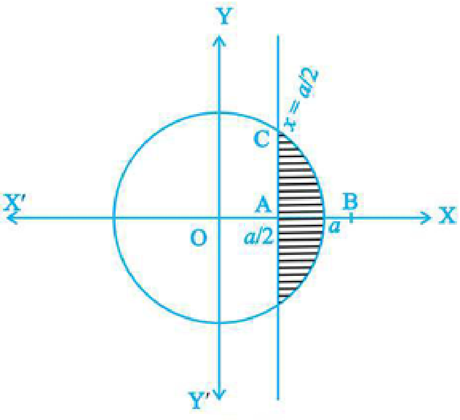


Fig. 8.9

Solving the equation $x^2 + y^2 = a^2$ and $x = \frac{a}{2}$., we obtain their points of intersection which are $\left(\frac{a}{2}, \sqrt{3}\frac{a}{2}\right)$ and $\left(\frac{a}{2}, -\frac{\sqrt{3}a}{2}\right)$.

Hence, from Fig. 8.9, we get

Required Area $= 2\,Area\,of\,OAB = 2\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}\,dx$

$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{2}}^{a}$

$= 2\left[\frac{a^2}{2}.\frac{\pi}{2} - \frac{a}{4}.a\frac{\sqrt{3}}{2} - \frac{a^2}{2}.\frac{\pi}{6}\right]$

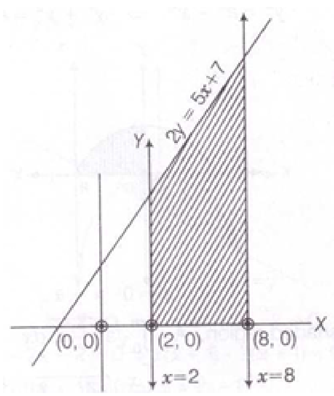
$= \frac{a^2}{12}\left(6\pi - 3\sqrt{3} - 2\pi\right)$

$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right) sq\,units.$

10 **Using integration, find the area of the region bounded by the line $2y = 5x + 7$, $x - axis$ and the lines $x = 2$ and $x = 8$.**

We have $2y = 5x + 7$

$\Rightarrow y = \frac{5x}{2} + \frac{7}{2}$

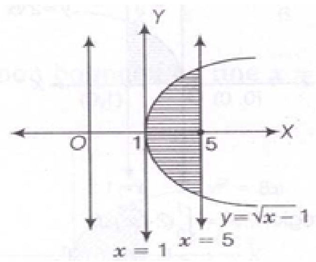


$\therefore$ Area of shaded region=

$\frac{1}{2}\int_2^8 (5x + 7)dx = \frac{1}{2}\left[5.\frac{x^2}{2} + 7x\right]_2^8$

$= \frac{1}{2}[5\cdot 32 + 7\cdot 8 - 10 - 14] = \frac{1}{2}[160 + 56 - 24]$

$= \frac{192}{2} = 96\,sq\,units$

11	**Draw a rough sketch of the curve $y=\sqrt{x-1}$ in the interval $[1,\ 5]$. Find the area under the curve and between the lines $x=1$ *and* $x=5$.** Given equation of the curve is $y=\sqrt{x-1}$ $\Rightarrow y^2 = x-1$  $\therefore$ Area of shaded region, $A=\int_1^5 (x-1)^{1/2}dx = \left[\frac{2.(x-1)^{3/2}}{3}\right]_1^5$ $=\left[\frac{2}{3}\cdot(5-1)^{3/2}-0\right]=\frac{16}{3} sq\,unit$