	BCM SCHOOL BASANT AVENUE DUGRI ROAD LUDHIANA **ANSWER KEY (INTEGRALS)** **CLASS XII SC**
1	**d**
2	**a**
3	**d**
4	**Now integrate** $\int (\boldsymbol{tan\,3x + tan\,2x + tan\,x\,dx})$
5	$\text{Let } I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} \mathrm{dx} \ldots (i)$ $\text{Applying property } \int_0^a f(x)\mathrm{dx} = \int_0^a f(a-x)\mathrm{dx}, \text{ we get}$ $I = \int_0^{2\pi} \frac{\mathrm{dx}}{1+e^{\sin(2\pi - x)}} = \int_0^{2\pi} \frac{\mathrm{dx}}{1+e^{-\sin x}} = \int_0^{2\pi} \frac{\mathrm{dx}}{1+\frac{1}{e^{\sin x}}}$ $I = \int_0^{2\pi} \frac{e^{\sin x} dx}{e^{\sin x + 1}} \ldots (ii)$ $2I = \int_0^{2\pi} \frac{\mathrm{dx}}{1+e^{\sin x}} + \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} = \int_0^{2\pi} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx$ $= \int_0^{2x} \mathrm{dx} = [x]_0^{2\pi}$

6		
6	$\int \sqrt{\tan x}\,dx$ Let $t^2 = \tan x$ $\Rightarrow 2tdt = \sec^2 x dx$ $\Rightarrow dx = \frac{2tdt}{\sec^2 x}$ $\Rightarrow dx = \frac{2tdt}{1+t^4}$ $= \int \frac{2t^2}{1+t^4}dt$ $= \int \frac{(1+t^2-(1-t^2))}{1+t^4}dt$ $= \int \frac{1+t^2}{1+t^4}dt - \int \frac{1-t^2}{1+t^4}dt$ $= \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$ $= \int \frac{1+\frac{1}{t^2}}{(t+\frac{1}{t})^2+2}dt + \int \frac{1-\frac{1}{t^2}}{(t-\frac{1}{t})^2+2}dt$	Let $z = t - \frac{1}{t}$ $dz = (1+\frac{1}{t^2})dt$ and let $u = t + \frac{1}{t}$ $du = (1-\frac{1}{t^2})dt$ $= \int \frac{dz}{z^2+(\sqrt{2})^2} + \int \frac{du}{u^2-(\sqrt{2})^2}$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\log\left\|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right\| + C$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\log\left\|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right\| + C$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\log\left\|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right\| + C$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left\|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right\| + C$ $\therefore \int \sqrt{\tan x}\,dx$

7	$I=\int \frac{dx}{\sin x+\sin 2x}$ $=\int \frac{1}{\sin x+2\sin\cos x}dx$ $=\int \frac{1}{\sin x(1+2\cos x)}dx$ $=\int \frac{\sin x}{\sin^2 x(1+2\cos x)}dx$ Let u = cosx ⇒ du = −sinx dx Also, $\sin^2 x=1-\cos^2 x$ $=1-u^2$ $\therefore I=\int -\frac{1}{(1-u^2)(1+2u)}du$ $=\int \frac{1}{(1+u)(1-u)(1+2u)}du$ **Using partial fractions, we get** $\frac{1}{(1+u)(1-u)(1+2u)}=\frac{A}{1+u}+\frac{B}{1-u}+\frac{C}{1+2u}$
8	$\int \frac{\sin x}{\sin 3x}dx=\int \frac{\sin x}{3\sin x-4\sin^3 x}dx$ $=\int \frac{dx}{3-4\sin^2 x}$ $=\int \frac{\sec^2 x\, dx}{3\sec^2 x-4\tan^2 x}$ $=\int \frac{\sec^2 x\, dx}{3-\tan^2 x}$ $=-\int \frac{dt}{t^2-3}$ Let $\tan x=t$ $\sec^2 x dx=dt$ $=-\frac{1}{2\sqrt{3}}\log\left\|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right\|+c$

9	$$Let I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx \ldots\ldots\ldots\ldots (1)$$ Then $$I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)} + 2^{\cos\left(\frac{\pi}{2}-x\right)}} dx \quad \left[\int_0^a f(x)dx = \int_0^a f(a-x)dx\right.$$ $$I = \int_0^{\frac{\pi}{2}} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} dx \ldots\ldots\ldots (2)$$ Adding (1) and (2), we get $$2I = \int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} dx$$ $$2I = \int_0^{\frac{\pi}{2}} dx$$ $$2I = x_0^{\frac{\pi}{2}}$$ $$2I = \frac{\pi}{2}$$