

ANSWER KEY OF XI  
LIMITS AND DERIVATIVE

1	d
2	d
3	$\frac{dy}{dx} = \frac{-2}{(1 + \sin 2x)}$
4	$\lim_{x \rightarrow 4} \frac{ 4-x }{x-4}$ <p>L.H.L. <math>\lim_{x \rightarrow 4^-} \frac{-(4-x)}{x-4} = \lim_{x \rightarrow 4^-} \frac{-(4-x)}{-(4-x)} = 1</math></p> <p>R.H.L. <math>\lim_{x \rightarrow 4^+} \frac{4-x}{x-4} = \lim_{x \rightarrow 4^+} \frac{-(x-4)}{(x-4)} = -1</math></p> <p>L.H.L. <math>\neq</math> R.H.L.</p> <p><math>\therefore \lim_{x \rightarrow 4} \frac{ 4-x }{x-4}</math> does not exist</p> <p style="text-align: right; font-weight: bold;">does not exist</p>
5	$y = \frac{(x+2)(3x-1)}{(2x+5)}$ $\frac{dy}{dx} = \frac{d}{dx} \frac{(x+2)(3x-1)}{(2x+5)}$ $= \frac{(2x+5) \frac{d}{dx} (x+2)(3x-1) - (x+2)(3x-1) \frac{d}{dx} (2x+5)}{(2x+5)^2}$ $= \frac{(2x+5) \left[ (x+2) \frac{d}{dx} (3x-1) + (3x-1) \frac{d}{dx} (x+2) \right] - (x+2)(3x-1)[2+0]}{(2x+5)^2}$ $= \frac{(2x+5) [(x+2) \times 3 + (3x-1) \times 1] - 2[3x^2 + 6x - x - 2]}{(2x+5)^2}$ $= \frac{(2x+5)[3x+6+3x-1] - 6x^2 - 12x + 2x + 4}{(2x+5)^2}$ $= \frac{12x^2 + 30x + 10x + 25 - 6x^2 - 10x + 4}{(2x+5)^2}$ $= \frac{6x^2 + 30x + 29}{(2x+5)^2}$

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$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} \\
&= \lim_{h \rightarrow 0} \frac{(a^2 + 2ah + h^2) \sin(a+h) - a^2 \sin a}{h} \\
&= \lim_{h \rightarrow 0} \frac{a^2 \sin(a+h) + 2ah \sin(a+h) + h^2 \sin(a+h) - a^2 \sin a}{h} \\
&= \lim_{h \rightarrow 0} \frac{a^2 [\sin(a+h) - \sin a] + 2ah \sin(a+h) + h^2 \sin(a+h)}{h} \\
&= \lim_{h \rightarrow 0} \frac{a^2 2 \cos \left[ \frac{2a+h}{2} \right] \sin \frac{h}{2}}{2 \frac{h}{2}} + \lim_{h \rightarrow 0} 2a \sin(a+h) + \lim_{h \rightarrow 0} h \sin(a+h) \\
&= a^2 \cos \left[ \frac{2a+0}{2} \right] \times 1 + 2a \sin[a+0] + 0 \times \sin a \\
&= a^2 \cos a + 2a \sin a
\end{aligned}$$

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(i)  $(2x - 7)(3x + 5)^2(30x - 43)$ 

$$\begin{aligned}
& \lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y} \\
&= \lim_{y \rightarrow 0} \frac{x \sec(x+y) + y \sec(x+y) - x \sec x}{y} \\
&= \lim_{y \rightarrow 0} \frac{[x \sec(x+y) - x \sec x]}{y} + \lim_{y \rightarrow 0} \frac{y \sec(x+y)}{y} \\
&= \lim_{y \rightarrow 0} \frac{x [\sec(x+y) - \sec x]}{y} + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \lim_{y \rightarrow 0} \frac{x \left[ \frac{1}{\cos(x+y)} - \frac{1}{\cos x} \right]}{y} + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \lim_{y \rightarrow 0} x \left[ \frac{\cos x - \cos(x+y)}{y \cdot \cos(x+y) \cdot \cos x} \right] + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \lim_{y \rightarrow 0} \frac{x \left[ \frac{-2 \sin \left( \frac{x+x+y}{2} \right)}{\sin \left( \frac{x-x-y}{2} \right)} \right]}{y \cos(x+y) \cdot \cos x} + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \frac{x [-2 \sin(x + \frac{y}{2}) \cdot \sin(-\frac{y}{2})]}{\cos(x+y) \cdot \cos x \cdot y} + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \lim_{\substack{y \rightarrow 0 \\ \therefore y \rightarrow 0}} x \left[ \frac{[2 \sin(x + \frac{y}{2}) \sin(\frac{y}{2})]}{\cos(x+y) \cdot \cos x \cdot (\frac{y}{2}) \cdot 2} \right] + \lim_{y \rightarrow 0} \sec(x+y)
\end{aligned}$$

 $\therefore$  Taking the limits we have

$$= x \left[ \sin x \cdot \frac{1}{\cos x \cdot \cos x} \right] + \sec x$$

(ii)  $= x \sec x \tan x + \sec x = \sec x (x \tan x + 1)$