

# BCM SCHOOL BASANT AVENUE DUGRI ROAD LUDHIANA

## ASSIGNMENT OF PROBABILITY

### ANSWER KEY

|          |  |
|----------|--|
| <b>1</b> | <b>(A) <math>\frac{4}{7}</math></b>  |
| <b>2</b> | <b>(c) <math>1 - P(A') P(B')</math></b>  |
| <b>3</b> | <b>(a) <math>P(E F) + P(E' F) = 1</math></b>   |
| <b>4</b> | <p> <math>P(E) = \frac{1}{6} P(\bar{E}) = \frac{5}{6}</math>, <math>P(F) = \frac{1}{12} P(\bar{F}) = \frac{11}{12}</math> A wins if he gets a total of 7 in 1st, 3rd or 5th ... throws.<br/> Probability of A getting a total of 7 in the 1st throw = <math>\frac{1}{6}</math> A will get the 3rd throw if he fails in the 1st throw and B fails in the 2nd throw.<br/> Probability of A getting a total of 7 in the 3rd throw = <math>P(\bar{E}) P(\bar{F}) P(E) = \frac{5}{6} \times \frac{11}{12} \times \frac{1}{6}</math> Similarly, probability of getting a total of 7 in the 5th throw =<br/> <math>P(\bar{E}) P(\bar{F}) P(E) = \frac{5}{6} \times \frac{11}{12} \times \frac{1}{6}</math><br/> <math>P(\bar{E}) P(\bar{F}) P(\bar{E}) P(\bar{F}) P(E) = \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{1}{6}</math> and so on<br/> Probability of winning of A = <math>\frac{1}{6} + \left( \frac{5}{6} \times \frac{11}{12} \times \frac{1}{6} \right) +</math><br/> <math>\left( \frac{5}{6} \times \frac{11}{12} \times \frac{5}{6} \times \frac{11}{12} \times \frac{1}{6} \right) + \dots = \frac{\frac{1}{6}}{1 - \frac{5}{6} \times \frac{11}{12}} = \frac{12}{17}</math><br/> <math>\therefore</math> Probability of winning of B = <math>1 - \text{Probability of winning of A} = 1 - \frac{12}{17} = \frac{5}{17}</math> </p> |

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Here, probability of  $P$  speaking truth,  $P(T) = 70\% = \frac{7}{10}$

$\therefore$  probability of  $P$  speaking lie,  $P(L) = 1 - \frac{7}{10} = \frac{3}{10}$

Probability of  $Q$  speaking truth,  $Q(T) = 80\% = \frac{4}{5}$

$\therefore$  probability of  $Q$  speaking lie,  $Q(L) = 1 - \frac{4}{5} = \frac{1}{5}$

Probability of both  $P$  and  $Q$  agree in same fact  $P(S) = P(T) \cap Q(T) + P(L) \cap Q(L)$

As, all these events are independent event,

$\therefore P(S) = P(T)Q(T) + P(L)Q(L)$

$$P(S) = \frac{7}{10} * \frac{4}{5} + \frac{3}{10} * \frac{1}{5} = \frac{28}{50} + \frac{3}{50} = \frac{31}{50}$$

So, percentage of both of them agree on same fact  $= \frac{31}{50} * 100 = 62\%$

Also, we can clearly see that when both of them agree, they also speak lie

and probability of that is  $\frac{3}{50}$ .

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The probability distribution of  $X$  is

|        |   |     |      |      |     |
|--------|---|-----|------|------|-----|
| $X$    | 0 | 1   | 2    | 3    | 4   |
| $P(X)$ | 0 | $k$ | $4k$ | $2k$ | $k$ |

The given distribution is a probability distribution.

$$\therefore \sum p_i = 1$$

$$\Rightarrow 0 + k + 4k + 2k + k = 1$$

$$\Rightarrow 8k = 1$$

$$\Rightarrow k = 0.125$$

$$(i) P(\text{getting admission in exactly one college}) = P(X = 1) = k = 0.125$$

$$(i) P(\text{getting admission in exactly one college}) = P(X = 1) = k = 0.125$$

$$(ii) P(\text{getting admission in at most 2 colleges}) = P(X \leq 2) = 0 + k + 4k = 5k = 0.625$$

$$(iii) P(\text{getting admission in atleast 2 colleges}) = P(X \geq 2) = 4k + 2k + k = 7k = 0.875 .$$

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Bag X = 4 white, 2 black.

Bag Y = 3 white, 3 black

Let A be the event of selecting one white and one black ball.

$E_1$  first bag selected

$E_2$  second bag selected

$$P(E_1) = \frac{1}{2} \quad P(E_2) = \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} = \frac{16}{30}$$

$$P(AE_2) = \frac{3}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{3}{6} \times \frac{3}{5} = \frac{18}{30}$$

$$P(E_2/A) = \frac{P(E_2) P(A/E_2)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$P(E_2/A) = \frac{\frac{1}{2} \times \frac{18}{30}}{\frac{1}{2} \times \frac{16}{30} + \frac{1}{2} \times \frac{18}{30}}$$

$$= \frac{18}{16 + 18}$$

$$= \frac{18}{34}$$

$$= \frac{9}{17}$$

The events are defined as follows:

E: Two ball drawn are white

A: There are 2 white balls in the bag

B: There are 3 white balls in the bag

C: There are 4 white balls in the bag

Then,  $P(A) = P(B) = P(C) = 1/3$

$$P\left(\frac{E}{A}\right) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$$

$$P\left(\frac{E}{B}\right) = \frac{{}^3C_2}{{}^4C_2} = \frac{3}{6} = \frac{1}{2}$$

$$P\left(\frac{E}{C}\right) = \frac{{}^4C_2}{{}^4C_2} = 1$$

$$\therefore \text{Required probability} = P\left(\frac{C}{E}\right)$$

Apply Baye's theorem:

$$\begin{aligned} P\left(\frac{C}{E}\right) &= \frac{P(C) \cdot P\left(\frac{E}{C}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)} \\ &= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1} = \frac{3}{5} \end{aligned}$$

Thus, the required probability is 3/5.

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$E_1$  : Loan at fixed rate

$E_2$  : Loan at floating rate

$E_3$  : Loan at variable rate A :

A person defaults on loan

$$P(E_1) = \frac{10}{100}; P(E_2) = \frac{20}{100}; P(E_3) = \frac{70}{100}$$

$$P(A/E_1) = \frac{5}{100}; P(A/E_2) = \frac{3}{100}; P(A/E_3) = \frac{1}{100}$$

$$(i) P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

$$= \frac{10}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{3}{100} + \frac{70}{100} \times \frac{1}{100}$$

$$= \frac{50+60+70}{10000}$$

$$\frac{180}{10000} = 0.018$$

$$(ii) P(E_3/A) = \frac{P(E_3)(P(A/E_3))}{P(A)}$$

$$= \frac{\frac{70}{100} \times \frac{1}{100}}{\frac{180}{10000}} = \frac{70}{10000 \times \frac{180}{10000}} = \frac{7}{18}$$