

$$\begin{aligned} \textcircled{1} \cdot P.E. &= mgh \\ &= \frac{mg(u^2 \sin^2 \theta)}{2g} \\ &= \left(\frac{1}{2} mu^2\right) \sin^2 \theta \\ &= E \sin^2 \theta. \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad u_x &= x; \quad u_y = y \\ R &= 2H \\ \frac{u^2 \sin 2\theta}{g} &= 2 \left(\frac{u^2 \sin^2 \theta}{2g} \right) \\ 2 \sin \theta \cos \theta &= \sin^2 \theta \\ 2 \cos \theta &= \sin \theta \\ 2 &= \tan \theta \\ 2 &= \frac{u_y}{u_x} \\ \boxed{\frac{y}{x} = 2} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \cdot \frac{u^2 \sin 2\theta}{g} &= 50 \quad (i) \\ \frac{u^2 \sin^2 \theta}{2g} &= 10 \quad (ii) \\ \text{Divide (i) by (ii):} \\ \frac{u^2 \cdot 2 \sin \theta \cos \theta}{g} \times \frac{2g}{u^2 \sin^2 \theta} &= \frac{50}{10} \\ \frac{4 \cos \theta}{\sin \theta} &= 5 \\ \frac{4}{5} &= \tan \theta \\ \boxed{\theta = \tan^{-1}(0.8)} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \text{1st ball.} \\ u &= 20 \text{ m/s}; \quad t = 1 \text{ s} \\ a &= -9.8 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} y_1 &= ut + \frac{1}{2} at^2 \\ &= 20(1) + \frac{1}{2} \times (-9.8) \times (1)^2 \\ &= 15.1 \text{ m} \end{aligned}$$

2nd ball.

$$\begin{aligned} y_2 &= u_y t + \frac{1}{2} a_y t^2 \\ &= (u \sin \theta) t + \frac{1}{2} \times (-9.8) \times (1)^2 \\ &= (20 \sin 30^\circ)(1) - 4.9 \\ &= 10 - 4.9 \\ &= 5.1 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Separation} &= 15.1 - 5.1 \\ &= 10 \text{ m} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad u &= 100 \text{ m/s.} \\ x &= u_x t + \frac{1}{2} a_x t^2 \\ &= (u \cos \theta) T \quad (\because a_x = 0) \\ v_y &= u_y + a_y t \\ 0 &= (u \sin \theta) - g t \\ t &= \frac{u \sin \theta}{g} = \frac{100 \times \sin 30^\circ}{9.8} \\ &= \frac{100 \times \frac{1}{2}}{9.8} = 5.1 \end{aligned}$$

$$T = 2t = 10.2 \text{ s}$$

$$\begin{aligned} x &= (u \cos \theta) T \\ &= 100 \times \frac{\sqrt{3}}{2} \times 10.2 \\ &= 883.3 \text{ m} \end{aligned}$$

$$\underline{6.} \quad v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$$

$$t = T = \frac{2u \sin \theta}{g}$$

$$v' = \sqrt{u^2 + g^2 \cdot \frac{4u^2 \sin^2 \theta}{g^2} - 2ug \left(\frac{2u \sin \theta}{g} \right) \sin \theta}$$

$$= \sqrt{u^2} = u$$

$$\text{Also, } \tan \beta = \frac{u \sin \theta - g \left(\frac{2u \sin \theta}{g} \right)}{u \cos \theta}$$

$$\tan \beta = -\frac{u \sin \theta}{u \cos \theta}$$

$$\tan \beta = -\tan \theta$$

$$\tan \beta = \tan(-\theta)$$

$$\boxed{\beta = -\theta}$$

$$R_{\max} = \frac{2}{\sqrt{3}} R$$

$$\frac{u^2}{g} = \left(\frac{2}{\sqrt{3}} \right) \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\sqrt{3}}{2} = 2 \sin \theta \cos \theta$$

$$\frac{\sqrt{3}}{2} = \sin 2\theta$$

$$\sin 60^\circ = \sin 2\theta$$

$$60^\circ = 2\theta$$

$$\boxed{\theta = 30^\circ}$$

$$y = \sqrt{3} x - \frac{g}{2} x^2$$

$$= (\tan \theta) x - \frac{g}{2u^2 \cos^2 \theta} x^2$$

$$\tan \theta = \sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$

$$u^2 \cos^2 \theta = 1$$

$$u^2 \cos^2 60^\circ = 1$$

$$u^2 \left(\frac{1}{4} \right) = 1$$

$$u^2 = 4$$

$$\boxed{u = 2} \text{ m/s.}$$

$$\underline{9.} \quad R_p = R_0$$

$$\frac{(\sqrt{2}v)^2 \sin 2(15^\circ)}{g} = \frac{v^2 \sin 2\theta}{g}$$

$$2 \cancel{v}^2 \sin 30^\circ = \cancel{v}^2 \sin 2\theta$$

$$1 = \sin 2\theta$$

$$\sin 90^\circ = \sin 2\theta$$

$$\boxed{\theta = 45^\circ}$$

10.

$$m \quad u_1 = 60 \text{ km/h} \quad h \quad R_1 = 400 \text{ m}$$

$$2m \quad u_2 = 30 \text{ km/h} \quad 4h \quad R_2 = ?$$

$$R = u \sqrt{\frac{2h}{g}}$$

$$\frac{R_2}{R_1} = \frac{u_2}{u_1} \sqrt{\frac{h_2}{h_1}}$$

$$\frac{R_2}{400} = \frac{30}{60} \sqrt{\frac{4h}{h}}$$

$$\frac{R_2}{400} = \frac{1}{2} \times 2$$

$$\boxed{R_2 = 400 \text{ m}}$$

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