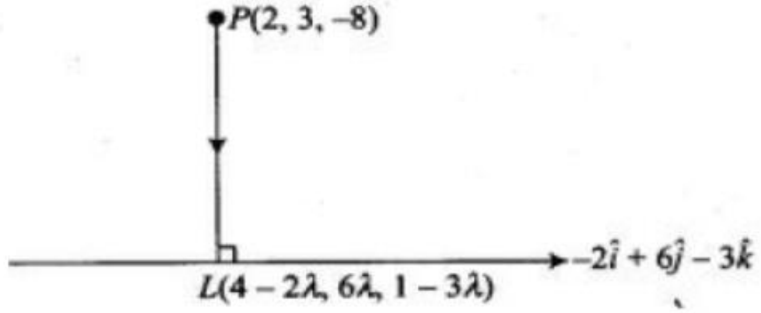
	ANSWER KEY OF **THREE-DIMENSIONAL GEOMETRY**
1	**(C)** $\sqrt{b^2+c^2}$
2	**(B)2**
3	**(d)** -1
4	**A)coincident**
5	(A) $\vec{r} = \lambda\,\hat{\imath}$
6	**(A)-1**
7	(A)
8	**(A)(2,0,0)**
9	$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda\ (say)$ x=3λ−1, y=5λ−3 and z=7λ−5 So, the coordinates of a general point on this line are (3λ−1, 5λ−3, 7λ−5). The equation of the second line is given below: $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu\ (say)$ x=μ+2, y=3μ+4 and z=5μ+6 So, the coordinates of a general point on this line are (μ+2, 3μ+4, 5μ+6). If the lines intersect, then they have a common point. So, for some values of λ and μ, we must have: 3λ−1=μ+2, 5λ−3=3μ+4 and 7λ−5=5μ+6 ⇒ 3λ−μ=3, 5λ−3μ=7 and 7λ−5μ=11 Solving the first two equations, 3λ−μ=3 and 5λ−3μ=7, we get: λ=1/2 and μ=−3/2 Since λ=1/2 and μ=−3/2 satisfy the third equation, 7λ−5μ=11, the given lines intersect each other. When λ=1/2 in (3λ−1, 5λ−3, 7λ−5), the coordinates of the required point of intersection are (1/2, −1/2, −3/2)

11

We have, equation of line as (4-x)/2 = y/6 = (1-z)/3

$$\Rightarrow \quad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$$

$$\Rightarrow \quad x = -2\lambda + 4,\ y = 6\lambda \text{ and } z = -3\lambda + 1$$



Let the foot of perpendicular from point $P(2, 3, -8)$ on the line is $L(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$

Then the direction ratios of PL are proportional to $(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda + 8)$ or $(2 - 2\lambda, 6\lambda - 3, 9 - 3\lambda)$

Also, direction ratios of line are $-2, 6, -3$.

Since, PL is perpendicular to give line.

$$\therefore \quad -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(9 - 3\lambda) = 0$$

$$\Rightarrow \quad -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$$

$$\Rightarrow \quad 49\lambda = 49$$

$$\Rightarrow \quad \lambda = 1$$

So, the coordinates of L are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda) \equiv (2, 6, -2)$.

Also, length of $PL = \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2}$

$$= \sqrt{0 + 9 + 36} = 3\sqrt{5} \text{ units}$$

11

We have, equation of the line as (x+5)/1 = (y+3)/4 = (z-6)/-9 = λ

$$\Rightarrow \quad x = \lambda - 5,\ y = 4\lambda - 3,\ z = 6 - 9\lambda$$

Let the coordinates of L are $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$

Then direction ratios of PL are $(\lambda - 5 - 2, 4\lambda - 3 - 4, 6 - 9\lambda + 1)$ or $(\lambda - 7,\ 4\lambda - 7, 7 - 9\lambda)$.

Also, the direction ratios of given line are $1, 4, -9$.

Since, PL is perpendicular to the given line.

$$\therefore \quad (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (7 - 9\lambda) \cdot (-9) = 0$$

$$\Rightarrow \quad \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$$

$$\Rightarrow \quad 98\lambda = 98$$

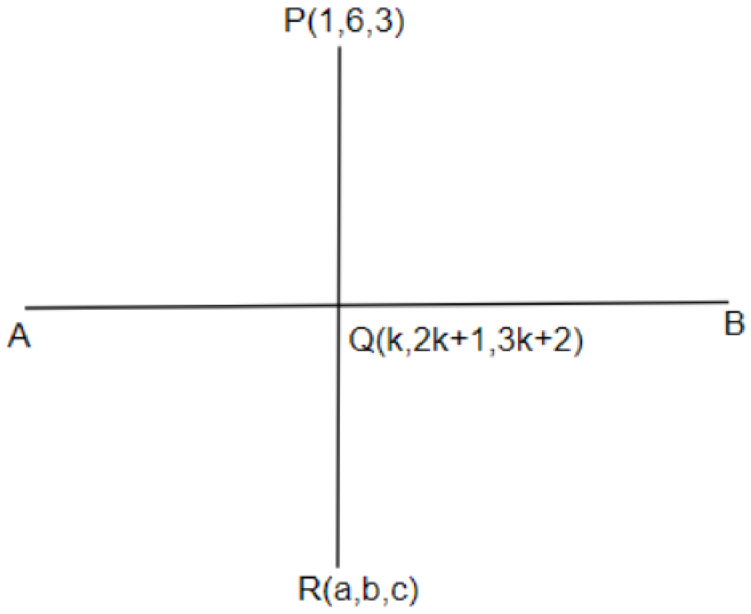
$$\Rightarrow \quad \lambda = 1$$

So, the coordinates of L are $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda) \equiv (-4, 1, -3)$.

$$\therefore \quad \text{Also } PL = \sqrt{(-4-2)^2 + (1-4)^2 + (-3+1)^2}$$

$$= \sqrt{36 + 9 + 4} = 7 \text{ units}$$

12	The correct option is **A** (1, 0, 7)



Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = k$

Then any point on the line AB is

$x = k,\ y = 2k + 1,\ z = 3k + 2$

Direction ratio of PQ = $(k - 1, 2k - 5, 3k - 1)$

$\because$ PQ is perpendicular to AB.

$\Rightarrow 1(k - 1) + 2(2k - 5) + 3(3k - 1) = 0$

$\Rightarrow k = 1$

$\therefore$ Coordinates of Q is (1, 3, 5).

Also, Q is the mid-point of PR.

$\Rightarrow \frac{a+1}{2} = 1,\ \frac{b+6}{2} = 3,\ \frac{c+3}{2} = 5$

$\Rightarrow a = 1,\ b = 0,\ c = 7$

$\therefore$ The image of (1, 6, 3) is (1, 0, 7).

13	x = py + q, z = ry + s $\Rightarrow \quad y = \frac{x-q}{p}$ and $y = \frac{z-s}{r}$ $\Rightarrow \quad \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r} \quad$ (i) Similarly line $x = p'y + q', z = r'y + s'$ $\Rightarrow \quad \frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'} \quad$ (ii) Line (i) is parallel to the vector $p\hat{i} + \hat{j} + r\hat{k}$. Line (ii) is parallel to the vector $p'\hat{i} + \hat{j} + r'\hat{k}$. Line are perpendicular, $\therefore \quad (p\hat{i} + \hat{j} + r\hat{k}) \cdot (p'\hat{i} + \hat{j} + r'\hat{k})$ $\therefore \quad pp' + 1 + rr' = 0.$
14	Here $\vec{a_1} = -3\hat{i} + \hat{j} + 5\hat{k}, \vec{b_1} = -3\hat{i} + \hat{j} + 5\hat{k}, \vec{a_2} = -\hat{i} + 2\hat{j} + 5\hat{k}, \vec{b_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$ Now $(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 2(-5) - 1(-15 + 5) = -10 + 10 = 0$ $\therefore$ Given lines are coplanar.