

①. P.E. = mgh

$$= \frac{mg(u^2 \sin^2 \theta)}{2g}$$

$$= \left(\frac{1}{2} mu^2\right) \sin^2 \theta$$

$$= E \sin^2 \theta.$$

②. $u_x = x$; $u_y = y$
 $R = 2H$

$$\frac{u^2 \sin 2\theta}{g} = 2 \left(\frac{u^2 \sin^2 \theta}{2g} \right)$$

$$2 \sin \theta \cos \theta = \sin^2 \theta$$

$$2 \cos \theta = \sin \theta$$

$$2 = \tan \theta$$

$$2 = \frac{u_y}{u_x}$$

$\frac{y}{x} = 2$

③.
$$\frac{u^2 \sin 2\theta}{g} = 50 \quad (i)$$

$$\frac{u^2 \sin^2 \theta}{2g} = 10 \quad (ii)$$

Divide (i) by (ii);

$$\frac{u^2 \cdot 2 \sin \theta \cos \theta}{g} \times \frac{2g}{u^2 \sin^2 \theta} = \frac{50}{10}$$

$$\frac{4 \cos \theta}{\sin \theta} = 5$$

$$\frac{4}{5} = \tan \theta$$

$\theta = \tan^{-1}(0.8)$

4. 1st ball.
 $u = 20 \text{ m/s}$; $t = 1 \text{ s}$
 $a = -9.8 \text{ m/s}^2$

$$y_1 = ut + \frac{1}{2} at^2$$

$$= 20(1) + \frac{1}{2} \times (-9.8) \times (1)^2$$

$$= 15.1 \text{ m}$$

2nd ball.

$$y_2 = u_y t + \frac{1}{2} a_y t^2$$

$$= (u \sin \theta) t + \frac{1}{2} \times (-9.8) \times (1)^2$$

$$= (20 \sin 30^\circ)(1) - 4.9$$

$$= 10 - 4.9$$

$$= 5.1 \text{ m}$$

Separation $15.1 - 5.1$
 $= 10 \text{ m}$

5. $u = 100 \text{ m/s}$.

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$= (u \cos \theta) T \quad (\because a_x = 0)$$

$$v_y = u_y + a_y t$$

$$0 = (u \sin \theta) - g t$$

$$t = \frac{u \sin \theta}{g} = \frac{100 \times \sin 30^\circ}{9.8}$$

$$= \frac{100 \times \frac{1}{2}}{9.8} = 5.10$$

$T = 2t = 10.2 \text{ s}$

$$x = (u \cos \theta) T$$

$$= 100 \times \frac{\sqrt{3}}{2} \times 10.2$$

$$= 883.3 \text{ m}$$

6. $v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$

$t = T = \frac{2u \sin \theta}{g}$

$v' = \sqrt{u^2 + g \cdot \frac{4u^2 \sin^2 \theta}{g^2} - 2ug \left(\frac{2u \sin \theta}{g} \right) \sin \theta}$

$= \sqrt{u^2} = u$

Also, $\tan \beta = \frac{u \sin \theta - g \left(\frac{2u \sin \theta}{g} \right)}{u \cos \theta}$

$\tan \beta = \frac{-u \sin \theta}{u \cos \theta}$

$\tan \beta = -\tan \theta$

$\tan \beta = \tan(-\theta)$

$\beta = -\theta$

$R_{\max} = \frac{2}{\sqrt{3}} R$

$\frac{u^2}{g} = \left(\frac{2}{\sqrt{3}} \right) \frac{u^2 \sin 2\theta}{g}$

$\frac{\sqrt{3}}{2} = 2 \sin \theta \cos \theta$

$\frac{\sqrt{3}}{2} = \sin 2\theta$

$\sin 60^\circ = \sin 2\theta$

$60^\circ = 2\theta$

$\theta = 30^\circ$

$y = \sqrt{3}x - \frac{g}{2}x^2$

$y = (\tan \theta)x - \frac{g}{2u^2 \cos^2 \theta}x^2$

$\tan \theta = \sqrt{3}$

$\theta = 60^\circ$

$u^2 \cos^2 \theta = 1$

$u^2 \cos^2 60^\circ = 1$

$u^2 \left(\frac{1}{4} \right) = 1$

$u^2 = 4$

$u = 2 \text{ m/s}$

9. $R_p = R_q$

$\frac{(\sqrt{2}v)^2 \sin 2(15^\circ)}{g} = \frac{v^2 \sin 2\theta}{g}$

$2v^2 \sin 30^\circ = v^2 \sin 2\theta$

$1 = \sin 2\theta$

$\sin 90^\circ = \sin 2\theta$

$\theta = 45^\circ$

10.

m $u_1 = 60 \text{ km/h}$ h $R_1 = 400 \text{ m}$

2m $u_2 = 30 \text{ km/h}$ 4h $R_2 = ?$

$R = u \sqrt{\frac{2h}{g}}$

$\frac{R_2}{R_1} = \frac{u_2}{u_1} \sqrt{\frac{h_2}{h_1}}$

$\frac{R_2}{400} = \frac{30}{60} \sqrt{\frac{4h}{h}}$

$\frac{R_2}{400} = \frac{1}{2} \times 2$

$R_2 = 400 \text{ m}$

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