1	$T_n = (2n+1)^3 - (2n)^3 = (2n+1-2n)[(2n+1)^2 + (2n+1)2n + (2n)^2]$ $= 12n^2 + 6n + 1$ (i) Sum of n terms, $S_n = \sum_{n=1}^{n}(12n^2+6n+1) = 12\cdot\frac{n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} + n$ $= 2n(n+1)(2n+1) + 3n(n+1) + n$ $= 2n(2n^2+3n+1) + 3n^2 + 3n + n$ $= 4n^3 + 9n^2 + 6n$ (ii) Sum of 10 terms, $S_{10} = 4\times(10)^3 + 9\times(10)^2 + 6\times 10$ $= 4000 + 900 + 60 = 4960$
2	$2y = \frac{x+3z}{2}$ $\Rightarrow \quad 4y = x + 3z \quad$ (i) Also, x, y and z are in G.P. Therefore, $y = xr$ and $z = xr^2$, where 'r' is the common ratio. $\therefore \quad 4xr = x + 3xr^2 \quad$ [Using (i)] $\Rightarrow \quad 4r = 1 + 3r^2 \Rightarrow 3r^2 - 4r + 1 = 0 \Rightarrow (3r-1)(r-1) = 0$ $\Rightarrow \quad r = \frac{1}{3}$ (For $r = 1$; x, y, z are not distinct)
3	On subtracting Eq. (ii) from Eq. (i), we get $0 = (2 + 1 + 3 + 5 + 7 + \ldots$ up to 50 terms$) - t_{50}$ $\Rightarrow \quad t_{50} = 2 + [1 + 3 + 5 + 7 + \ldots$ upto 49 terms$]$ $= 2 + \frac{49}{2}[2\times 1 + (49-1)\times 2] = 2 + 49(1+48) = 2 + 49^2$
4	$\therefore \quad$ Volume $= \frac{a}{r}\times a\times ar = 216\ \text{cm}^3$ $\Rightarrow \quad a^3 = 216 = 6^3 \Rightarrow a = 6$ Also, Surface area $= 2\left(\frac{a}{r}\cdot a + a\cdot ar + \frac{a}{r}\cdot ar\right) = 252$ $\Rightarrow \quad 2a^2\left(\frac{1}{r} + r + 1\right) = 252 \Rightarrow 2\times 36\left(\frac{1+r^2+r}{r}\right) = 252$ $\Rightarrow \quad 2(1 + r^2 + r) = 7r \Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow (2r-1)(r-2) = 0$ $\therefore \quad r = \frac{1}{2}, 2$ For $r = \frac{1}{2}$: Length $= \frac{a}{r} = \frac{6\times 2}{1} = 12$, Breadth $= a = 6$, Height $= ar = 6\times\frac{1}{2} = 3$ For $r = 2$: Length $= \frac{a}{r} = \frac{6}{2} = 3$, Breadth $= a = 6$, Height $= ar = 6\times 2 = 12$

5

Let $S = 1^2 + 2^2 + 3^2 + \ldots + n^2$.

We have, $n^3 - (n-1)^3 = 3n^2 - 3n + 1$;

and by changing n into $n - 1$,

$$(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1;$$
$$(n-2)^2 - (n-3)^3 = 3(n-2)^2 - 3(n-2) + 1;$$

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$$3^3 - 2^3 = 3.3^2 - 3.3 + 1;$$
$$2^3 - 1^2 = 3.2^2 - 3.2 + 1;$$
$$1^2 - 0^2 = 3.1^2 - 3.1 + 1$$

Hence, by addition,

$$n^3 = 3(1^2 + 2^2 + 3^2 + \ldots + n^2) - 3(1 + 2 + 3 + \ldots + n) + n$$
$$= 3S - \frac{3n(n+1)}{2} + n$$
$$\Rightarrow \quad 3S = n^3 - n + \frac{3n(n+1)}{2} = n(n+1)\left(n - 1 + \frac{3}{2}\right)$$
$$\Rightarrow \quad S = \frac{n(n+1)(2n+1)}{6}$$

6

$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1;$$
$$(n-1)^4 - (n-2)^4 = 4(n-1)^3 - 6(n-1)^2 + 4(n-1) - 1;$$
$$(n-2)^4 - (n-3)^4 = 4(n-2)^3 - 6(n-2)^2 + 4(n-2) - 1;$$

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$$3^4 - 2^4 = 4.3^3 - 6.3^2 + 4.3 - 1;$$
$$2^4 - 1^4 = 4.2^3 - 6.2^2 + 4.2 - 1;$$
$$1^4 - 0^4 = 4.1^3 - 6.1^2 + 4.1 - 1.$$

Hence, by addition,

$$n^4 = 4S - 6(1^2 + 2^2 + \ldots + n^2) + 4(1 + 2 + \ldots + n) - n;$$
$$\therefore \quad 4S = n^4 + n + 6(1^2 + 2^2 + \ldots + n^2) - 4(1 + 2 + \ldots + n)$$
$$= n^4 + n + n(n+1)(2n+1) - 2n(n+1)$$
$$= n(n+1)(n^2 - n + 1 + 2n + 1 - 2)$$
$$= n(n+1)(n^2 + n);$$

$$\therefore \quad S = \frac{n^2(n+1)^2}{4} = \left\{\frac{n(n+1)}{2}\right\}^2$$

7	From the formula, $a_n = 1^2 + 2^2 + 3^2 + --- + n^2$ $a_n = \frac{n(n+1)(2n+1)}{6}$ $S_n = \frac{1}{6}[2\sum_{k-1}^{n} k^3 + 3\sum_{k-1}^{n} k^2 + \sum k$ $= \frac{1}{6}[2.\frac{n^2(n+1)^2}{4} + \frac{3.n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}]$ $= \frac{n(n+1)}{12}[n(n+1+(2n+1)+1]$ $= \frac{n(n+1)}{12}(n^2+n+2n+1+1)$ $= \frac{n(n+1)(n^2+3n+2)}{12}$ $= \frac{n(n+1)^2(n+2)}{12}$
8	**Ans:** $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$ $\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$ by C and D $\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$ $\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$ $\frac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$ by C and D $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$ Sq both side $\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$
9	Given that, pth term $= q \Rightarrow ar^{p-1} = q$ ...(i) and qth term $= p \Rightarrow ar^{q-1} = p$ ...(ii) On dividing Eq. (i) by Eq. (ii), we get $\frac{ar^{p-1}}{ar^{q-1}} = \frac{q}{p} \Rightarrow r^{p-q} = \frac{q}{p} \Rightarrow r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$ On substituting the value of r in Eq. (i), we get $a\left(\frac{q}{p}\right)^{\frac{p-1}{p-q}} = q \Rightarrow a = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$ $(p+q)$th term, $T_{p+q} = a \cdot r^{p+q-1}$ $= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}\left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}} = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}-\frac{p+q-1}{p-q}}$ $= q\left(\frac{q}{p}\right)^{\frac{q}{p-q}} = \frac{q^{\frac{q}{p-q}+1}}{p^{\frac{q}{p-q}}} = \frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}} = \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$