

	BCM SCHOOL BASNT AVENUE DUGRI ROAD LUDHINANA ANSWER KEY OF STRAIGHT LINES ASSIGNMENT CLASS XI
1	C
2	B
3	D
4	A
5	D

Equation of line having a and b intercepts on the axis is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$$

Given that $a + b = 14$

$$\Rightarrow b = 14 - a$$

$$\Rightarrow \frac{x}{a} + \frac{y}{14 - a} = 1$$

If equation (ii) passes through the point (3, 4) then

$$\frac{3}{a} + \frac{4}{14 - a} = 1$$

$$\Rightarrow \frac{3(14 - a) + 4a}{a(14 - a)} = 1$$

$$\Rightarrow 42 + a = 14a - a^2$$

$$\Rightarrow a^2 + a - 14a + 42 = 0$$

$$\Rightarrow a^2 - 13a + 42 = 0$$

$$\Rightarrow a^2 - 7a - 6a + 42 = 0$$

$$\Rightarrow a(a - 7) - 6(a - 7) = 0$$

$$\Rightarrow (a - 6)(a - 7) = 0$$

$$\Rightarrow a = 6, 7$$

$$\therefore b = 14 - 6 = 8, b = 14 - 7 = 7$$

Hence, the required equation of lines are $\frac{x}{6} + \frac{y}{8} = 1$

$$\Rightarrow 4x + 3y = 24$$

And $\frac{x}{7} + \frac{y}{7} = 1$

$$\Rightarrow x + y = 7$$

Step 1: Determine the equation of the Line

Let A = $(\cos \theta, \sin \theta)$ and

B = $(\cos \phi, \sin \phi)$ be the given points.

Then equation of AB is of the form,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - \sin \theta = \frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} (x - \cos \theta)$$

$$y(\cos \phi - \cos \theta) - \sin \theta(\cos \phi - \cos \theta) = x(\sin \phi - \sin \theta) - \cos \theta(\sin \phi - \sin \theta)$$

$$y(\cos \phi - \cos \theta) - \sin \theta \cos \phi + \sin \theta \cos \theta = x(\sin \phi - \sin \theta) - \cos \theta \sin \phi + \sin \theta \cos \theta$$

$$x(\sin \phi - \sin \theta) - y(\cos \phi - \cos \theta) + \sin \theta \cos \phi - \cos \theta \sin \phi = 0$$

$$x(\sin \phi - \sin \theta) - y(\cos \phi - \cos \theta) + \sin(\theta - \phi) = 0$$

$$[\because \sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi]$$

Step 2: Determine the distance

We know that distance of this line from the origin,

$$D = \left| \frac{0 - 0 + \sin(\theta - \phi)}{\sqrt{(\sin \phi - \sin \theta)^2 + (\cos \phi - \cos \theta)^2}} \right|$$

$$= \left| \frac{\sin(\theta - \phi)}{\sqrt{2 - 2(\cos \theta \cos \phi + \sin \theta \sin \phi)}} \right|$$

$$= \left| \frac{\sin(\theta - \phi)}{\sqrt{2} \sqrt{1 - \cos(\theta - \phi)}} \right|$$

$$= \left| \frac{\sin(\theta - \phi)}{\sqrt{2} \cdot \sqrt{2 \sin^2 \left(\frac{\theta - \phi}{2} \right)}} \right|$$

$$[\because 1 - \cos x = 2 \sin^2 \frac{x}{2}, \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}]$$

$$= \left| \frac{2 \sin \left(\frac{\theta - \phi}{2} \right) \cdot \cos \left(\frac{\theta - \phi}{2} \right)}{\sqrt{2} \sqrt{2} \cdot \sin \left(\frac{\theta - \phi}{2} \right)} \right|$$

$$= \left| \cos \left(\frac{\theta - \phi}{2} \right) \right|$$

$$= \left| \cos \left(\frac{\theta - \phi}{2} \right) \right|$$

Hence, the required perpendicular distance is $\left| \cos \left(\frac{\theta - \phi}{2} \right) \right|$.

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$$y = m_1x + c_1 \quad \dots(1)$$

$$y = m_2x + c_2 \quad \dots(2)$$

$$y = m_3x + c_3 \quad \dots(3)$$

On subtracting equation (1) from (2), we obtain

$$0 = (m_2 - m_1)x + (c_2 - c_1)$$

$$= (m_1 - m_2)x = c_2 - c_1$$

$$= x = \frac{c_2 - c_1}{m_1 - m_2}$$

On substituting this value of x in (1), we obtain

$$y = m_1 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_1$$

$$y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$$

$$y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

$$\therefore \left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right) \text{ is the point of intersection of lines (1) and (2).}$$

It is given that lines (1), (2), and (3) are concurrent. Hence, the point of intersection of lines (1) and (2) will also satisfy equation (3).

$$= \frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3 \left(\frac{c_2 - c_1}{m_1 - m_1} \right) + c_3$$

$$= \frac{m_1c_2 - m_2c_1}{m_1 - m_2} = \frac{m_3c_2 - m_3c_1 + c_3m_1 - c_3m_2}{m_1 - m_2}$$

$$= m_1c_2 - m_2c_1 - m_3c_2 + m_3c_1 - c_3m_1 + c_3m_2 = 0$$

$$= m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

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Equation of the required line passing through (1, 5) and (α_1, β_1) is

$$y - 5 = \frac{\beta_1 - 5}{\alpha_1 - 1} (x - 1)$$

$$\text{or } y - 5 = \frac{\frac{222}{23} - 5}{\frac{26}{23} - 1} (x - 1)$$

$$\text{or } 107x - 3y - 92 = 0$$

which is the equation of required line.

1
0

Let P_1 and P_2 be the length of perpendiculars from $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1.$$

$$\therefore P_1 = \left| \frac{\frac{\sqrt{a^2 - b^2} \cos \theta}{a} + \frac{0 \times \sin \theta}{b} - 1}{\sqrt{\left(\frac{\cos \theta}{a}\right)^2 + \left(\frac{\sin \theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\frac{\sqrt{a^2 - b^2} \cos \theta - 1}{a}}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right|$$

$$P_2 = \left| \frac{\frac{-\sqrt{a^2 - b^2} \cos \theta}{a} + \frac{0 \times \sin \theta}{b} - 1}{\sqrt{\left(\frac{\cos \theta}{a}\right)^2 + \left(\frac{\sin \theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\frac{-\sqrt{a^2 - b^2} \cos \theta - 1}{a}}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right|$$

$$\text{Now } P_1 P_2 = \left| \frac{\frac{\sqrt{a^2 - b^2} \cos \theta - 1}{a}}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right| \left| \frac{\frac{-\sqrt{a^2 - b^2} \cos \theta - 1}{a}}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}} \right|$$

$$= \left| \left[\frac{\sqrt{a^2 - b^2} \cos \theta - 1}{a} \right] \left[\frac{\sqrt{a^2 - b^2} \cos \theta + 1}{a} \right] \right|$$

$$= \frac{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}{\left| \left[\frac{(a^2 - b^2) \cos^2 \theta}{a^2} - 1 \right] \right|} = \frac{\left| \left[\frac{(a^2 - b^2) \cos^2 \theta}{a^2} - 1 \right] \right|}{\frac{b^2 \cos^2 \theta + a^2 - a^2 \cos^2 \theta}{a^2 b^2}}$$

$$= \frac{|a^2 - (a^2 - b^2) \cos^2 \theta|}{\frac{a^2 - (a^2 - b^2) \cos^2 \theta}{b^2}}$$

$$= a^2 - (a^2 - b^2) \cos^2 \theta \times \frac{b^2}{a^2 - (a^2 - b^2) \cos^2 \theta}$$

$$= b^2.$$