	BCM SCHOOL LUDHIANA **Answer key CLASS XII**
1	**not defined**
2	3
3	Given, $\frac{dy}{dx} - 3y\cot x = \sin 2x$ ...(i) This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, here P = -3 cot x and Q = sin 2x $\therefore \quad \mathrm{I} = e^{\int \mathrm{P} dx} = e^{-3\int \cot x dx}$ $\Rightarrow \quad \mathrm{IF} = e^{-3\log\|\sin x\|} = e^{\log\|\sin x\|^{-3}} = \|\sin x\|^{-3}$ $\therefore$ The general solution of differential equation is given by $y \times \mathrm{IF} = \int (\mathrm{IF} \times Q) dx + C$ $\Rightarrow \quad y(\sin x)^{-3} = \int (\sin x)^{-3}(\sin 2x) dx + C$ $= \int \frac{2\sin x\cos x}{\sin^3 x} dx + C$ $\therefore \quad y(\sin x)^{-3} = \int \frac{2\cos x}{\sin^2 x} dx + C$ ...(i) Therefore,on putting sin x = t $\Rightarrow$ cos x dx = dt in Eq. (i), we get $y(\sin x)^{-3} = 2\int \frac{dt}{t^2} + C = 2 \times \frac{t^{-1}}{-1} + C$ $\Rightarrow \quad y(\sin x)^{-3} = -\frac{2}{t} + C$ $\Rightarrow \quad y(\sin x)^{-3} = \frac{-2}{\sin x} + C$ [put t = sin x] $\Rightarrow$ y = -2 $\sin^2$ x + C $\sin^3$ x ...(ii) Therefore,on putting $x = \frac{\pi}{2}$ and y = 2 in Eq. (ii), we get, $2 = -2\sin^2\frac{\pi}{2} + C\sin^3\frac{\pi}{2} \Rightarrow 2 = -2 \cdot 1 + C \cdot 1$ $\Rightarrow$ C = 4 $\therefore y = -2\sin^2 x + 4\sin^3 x$

4

$y dx + x \log\left|\frac{y}{x}\right| dy - 2x dy = 0$

$\Rightarrow \quad y dx = \left[2x - x \log\left|\frac{y}{x}\right|\right] dy$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{2x - x\log\left|\frac{y}{x}\right|}$

Now, let$F(x, y) = \frac{y}{2x - x\log\left|\frac{y}{x}\right|}$

On replace x by λx and y by λy both sides, we get

$F(\lambda x, \lambda y) = \frac{\lambda y}{2\lambda x - \lambda x \log\left|\frac{\lambda y}{\lambda x}\right|}$

$= \frac{\lambda y}{\lambda\left[2x - x\log\left|\frac{y}{x}\right|\right]}$

$\Rightarrow F(\lambda x, \lambda y) = \lambda^0 \frac{y}{2x - x\log\left|\frac{y}{x}\right|} = \lambda^0 F(x, y)$

So, the given differential equation is homogeneous.

On putting y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ in Eq. (i).

we get $v + x\frac{dv}{dx} = \frac{vx}{2x - x\log\left|\frac{vx}{x}\right|} = \frac{v}{2 - \log|v|}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v}{2 - \log|v|} - v = \frac{v - 2v + v\log|v|}{2 - \log|v|}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v + v\log|v|}{2 - \log|v|}$

$\Rightarrow \quad \frac{2 - \log|v|}{v\log|v| - v} dv = \frac{dx}{x}$

On integrating both sides, we get

$\int \frac{2 - \log|v|}{v(\log|y| - 1)} dv = \int \frac{dx}{x}$

On putting log |v| = t $\Rightarrow \frac{1}{v} dv = dt$

Then, $\int \frac{2-t}{t-1} dt = \log|x| + C$

$\Rightarrow \int \left(\frac{1}{t-1} - 1\right) dt = \log|x| + C$

$\Rightarrow \quad \log|t - 1| - t = \log|x| + C$

$\Rightarrow \quad \log|\log v - 1| - \log v = \log|x| + C$ [put t = log |v|]

$\Rightarrow \quad \log\left|\frac{\log v - 1}{v}\right| = \log|x| + C \left[\because \log m - \log n = \log\left(\frac{m}{n}\right)\right]$

$\Rightarrow \log\left|\frac{\log v - 1}{v}\right| - \log|x| = C \Rightarrow \log\left|\frac{\log v - 1}{vx}\right| = C$

$\therefore \quad \log\left|\frac{\log\frac{y}{x} - 1}{y}\right| = c \left[\because y = vx \Rightarrow v = \frac{y}{x}\right]$

5

. The given function is

y = e^{ax} [c_1 cos bx + c_2 sin bx] ......(i)

Differentiating both sides of equation (i) with respect to x, we get

$\frac{dy}{dx} = e^{ax}[-bc_1 \sin bx + bc_2 \cos bx] + [c_1 \cos bx + c_2 \sin bx]e^{ax} \cdot a$

$\frac{dy}{dx} = e^{ax}[(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx]$ ......(ii)

Differentiating both sides of equation (ii) with respect to x, we get

$\frac{d^2y}{dx^2} = e^{ax}[(bc_2 + ac_1)(-b \sin bx) + (ac_2 - bc_1)(b \cos bx)] +$

$[(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx]e^{ax} \cdot a$

$= e^{ax}\left[\left(a^2c_2 - 2abc_1 - b^2c_2\right)\sin bx + \left(a^2c_1 + 2abc_2 - b^2c_1\right)\cos bx\right]$

Substituting the values of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y in the given differential equation, we get

L.H.S. = $e^{ax}\left[a^2c_2 - 2abc_1 - b^2c_2\right)\sin bx + \left(a^2c_1 + 2abc_2 - b^2c_1\right)\cos bx\right]$ -

$-2ae^{ax}[(bc_2 + ac_1)\cos bx + (ac_2 - bc_1)\sin bx] + \left(a^2 + b^2\right)e^{ax}[c_1 \cos bx + c_2 \sin bx]$

= e^{ax}[$\left(a^2c_2 - 2abc_1 - b^2c_2 - 2a^2c_2 + 2abc_1 + a^2c_2 + b^2c_2\right)\sin bx$ +

$\left(a^2c_1 + 2abc_2 - b^2c_1 - 2abc_2 - 2a^2c_1 + a^2c_1 + b^2c_1\right)\cos bx$]

= $e^{ax}[0 \times \sin bx + 0 \cos bx] = e^{ax} \times 0 = 0$ = R.H.S

Hence, the given function is a solution of the given differential equation.

6	The given differential equation can be written as $\left[xy\sin\left(\frac{y}{x}\right)-x^2\cos\left(\frac{y}{x}\right)\right]dy$ = $\left[xy\cos\left(\frac{y}{x}\right)+y^2\sin\left(\frac{y}{x}\right)\right]dx$ or $\frac{dy}{dx}=\frac{xy\cos\left(\frac{y}{x}\right)+y^2\sin\left(\frac{y}{x}\right)}{xy\sin\left(\frac{y}{x}\right)-x^2\cos\left(\frac{y}{x}\right)}$ Dividing numerator and denominator on RHS by x^2, we get $\frac{dy}{dx}=\frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)+\left(\frac{y^2}{x^2}\right)\sin\left(\frac{y}{x}\right)}{\frac{y}{x}\sin\left(\frac{y}{x}\right)-\cos\left(\frac{y}{x}\right)}$ ......(i) Clearly, equation (i) is a homogeneous differential equation of the form $\frac{dy}{dx}=g\left(\frac{y}{x}\right)$. To solve it, we make the substitution y = vx .....(ii) or $\frac{dy}{dx}=v+x\frac{dv}{dx}$ or $v+x\frac{dv}{dx}=\frac{v\cos v+v^2\sin v}{v\sin v-\cos v}$ (using (i) and (ii)) or $x\frac{dv}{dx}=\frac{2v\cos v}{v\sin v-\cos v}$ or $\left(\frac{v\sin v-\cos v}{v\cos v}\right)dv=\frac{2dx}{x}$ Therefore, $\int\left(\frac{v\sin v-\cos v}{v\cos v}\right)dv=2\int\frac{1}{x}dx$ or $\int\tan v dv-\int\frac{1}{v}dv=2\int\frac{1}{x}dx$ or $\log\lvert\sec v\rvert-\log\lvert v\rvert=2\log\lvert x\rvert+\log\lvert C_1\rvert$ or $\log\left\lvert\frac{\sec v}{vx^2}\right\rvert=\log\lvert \mathrm{C}_1\rvert$ or $\frac{\sec v}{vx^2}=\pm C_1$ ......(iii) Replacing v by $\frac{y}{x}$ in equation (iii), we get $\frac{\sec\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)(x^2)}=C$ where, $\mathrm{C}=\pm\mathrm{C}_1$ or sec $\left(\frac{y}{x}\right)=\mathrm{C}xy$

7	It is given that (x - y)(dx + dy) = dx - dy ⇒ (x - y + 1)dy = (1 - x + y)dx $\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$ ......(i) Let x – y = t $\Rightarrow \frac{d}{dx}(x-y) = \frac{dt}{dx}$ $\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$ Now, let us substitute the value of x-y and $\frac{dy}{dx}$ in equation (i), we get, $1 - \frac{dt}{dx} = \frac{1-t}{1+t}$ $\Rightarrow \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t}\right)$ $\Rightarrow \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$ $\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$ $\Rightarrow \left(\frac{1+t}{t}\right)dt = 2dx$ $\Rightarrow \left(1 + \frac{1}{t}\right)dt = 2dx$ .....(ii) On integrating both side, we get, t + log|t| = 2x + C ⇒ (x - y) + log |x - y| = 2x + C ⇒ log|x – y| = x + y + C ....(iii) Now, y = -1 at x = 0 Then, equation (iii), we get, log 1 = 0 - 1 + C ⇒ C = 1 Substituting C = 1 in equation (iii), we get, log|x - y| = x + y + 1

8	$\frac{dy}{dx}=\frac{x^3-3xy^2}{y^3-3x^2y}$ ...(i) This is a homogeneous differential equation, so, put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ Then, Eq. (i) becomes $v+x\frac{dv}{dx}=\frac{x^3-3x(vx)^2}{(vx)^3-3x^2(vx)}$ $\Rightarrow v+x\frac{dv}{dx}=\frac{1-3v^2}{v^3-3v}$ $\Rightarrow x\frac{dv}{dx}=\frac{1-3v^2}{v^3-3v}-v$ $\Rightarrow x\frac{dv}{dx}=\frac{1-3v^2-v^4+3v^2}{v^3-3v}$ $\Rightarrow x\frac{dv}{dx}=\frac{1-v^4}{v^3-3v}$ $\Rightarrow \left(\frac{v^3-3v}{1-v^4}\right)dv=\frac{dx}{x}$ On integrating both sides, we get $\int\left(\frac{v^3-3v}{1-v^4}\right)dv=\int\frac{dx}{x}$ $\Rightarrow \int\frac{v^3}{1-v^4}dv-3\int\frac{v}{1-v^4}dv=\log x+\log C$ ...(ii) $\Rightarrow -\frac{1}{4}\log\left(1-v^4\right)-\frac{3}{4}\log\left\|\frac{1+v^2}{1-v^2}\right\|=\log x+\log C$ $\Rightarrow -\frac{1}{4}\log\left[\left(1-v^4\right)\left(\frac{1+v^2}{1-v^2}\right)^3\right]=\log(Cx)$ $\Rightarrow -\frac{1}{4}\log\left[\left(1-v^2\right)\left(1+v^2\right)\times\frac{\left(1+v^2\right)^3}{\left(1-v^2\right)^3}\right]=\log(Cx)$ $\Rightarrow \log\left[\frac{\left(1+v^2\right)^4}{\left(1-v^2\right)^2}\right]^{-1/4}=\log Cx$ $\Rightarrow \frac{\left(1+v^2\right)^4}{\left(1-v^2\right)^2}=(Cx)^{-4}$ $\Rightarrow \frac{\left(1+y^2/x^2\right)^4}{\left(1-y^2/x^2\right)^2}=\frac{1}{C^4x^4}$ $[\because y=vx]$ $\Rightarrow \frac{\left(x^2+y^2\right)^4}{x^4\left(x^2-y^2\right)^2}=\frac{1}{C^4x^4}$ $\Rightarrow \left(x^2-y^2\right)=C^2\left(x^2+y^2\right)^2$ [taking square root]
9	$\frac{dy}{dx}=x+y$