(ANWERS)

SUBJECT: MATHEMATICS — **MAX. MARKS : 80**

CLASS : X — **DURATION : 3 HRS**

General Instruction:

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 20 MCQs carrying 1 mark each.
3. **Section B** has 5 questions carrying 02 marks each.
4. **Section C** has 6 questions carrying 03 marks each.
5. **Section D** has 4 questions carrying 05 marks each.
6. **Section E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The solution of the following pair of equation is:
   $x - 3y = 2$, $3x - y = 14$
   (a) $x = 5, y = 1$ (b) $x = 2, y = 3$ (c) $x = 1, y = 2$ (d) $x = 1, y = 4$
   Ans: (a) $x = 5, y = 1$
   Given, equations are $x - 3y = 2$ …(i)
   and $3x - y = 14$ …(ii)
   Solving equations (i) and (ii), we get $y = 1$
   $x = 2 + 3y = 2 + 3 \times 1 = 5$
   Hence, $x = 5$ and $y = 1$

2. What is the positive real root of $64x^2 - 1 = 0$?
   (a) 1/8 (b) 1/4 (c) 1/2 (d) 1/6
   Answer: (a) 1/8,−1/8
   Given, $64x^2 - 1 = 0$
   $\Rightarrow (8x + 1)(8x - 1) = 0$
   $\Rightarrow 8x = -1, 8x = 1$
   $\Rightarrow x = 1/8, -1/8$
   Thus, positive root is 1/8

3. In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are
   (a) congruent but not similar (b) similar but not congruent
   (c) neither congruent nor similar (d) congruent as well as similar
   Ans: (b) similar but not congruent

4. The LCM of smallest two-digit composite number and smallest composite number is:
   (a) 12 (b) 4 (c) 20 (d) 44
   Ans: (c) 20
   Smallest 2-digit number = $10 = 2 \times 5$
   Smallest composite number = $4 = 2^2$
   LCM = $2^2 \times 5 = 20$

5. If cosec A = 13/12, then the value of $\frac{2\sin A - 3\cos A}{4\sin A - 9\cos A}$

(a) 4 (b) 5 (c) 6 (d) 3

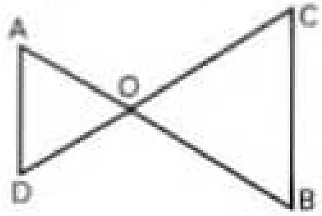
Ans: (d) 3

Given cosec A = 13/12,

$\sin A = \frac{12}{13}, \cos A = \frac{5}{13}$

$$Now, \frac{2\sin A - 3\cos A}{4\sin A - 9\cos A} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)} = \frac{24-15}{48-45} = \frac{9}{3} = 3$$

6. In the figure, if $\frac{OA}{OD} = \frac{OC}{OB}$, then



which pair of angles are equal? [1]

(a) ∠A = ∠C, ∠B = ∠D (b) ∠A = ∠B, ∠C = ∠D

(c) ∠C = ∠B, ∠A = ∠D (d) None of these

Ans: (a) ∠A = ∠C, ∠B = ∠D

In given figure, $\frac{OA}{OD} = \frac{OC}{OB} \Rightarrow \frac{OA}{OC} = \frac{OD}{OB}$

Then, in ΔAOD and ΔCOB

$\frac{OA}{OC} = \frac{OD}{OB}$ (proved above)

and ∠AOD = ∠COB (vertically opposite angles)

∴ ΔAOD ~ ΔCOB by SAS Similarity

Hence ∠A = ∠C and ∠B = ∠D. (Corresponding angles of similar triangles)

7. The value of 'a', if HCF (a, 18) = 2 and LCM (a, 18) = 36, is: (1)

(a) 2 (b) 5 (c) 7 (d) 4

Ans: (d) 4

Given, HCF (a, 18) = 2 and LCM (a, 18) = 36

HCF (a, b) × LCM (a, b) = a × b

⇒ 2 × 36 = a × 18 ⇒ a = 4

Hence, value of 'a' is 4.

8. If r = 3 is a root of quadratic equation $kr^2 - kr - 3 = 0$, then the value of k is:

(a) 1/2 (b) 3 (c) 1/3 (d) 1/4

Ans: (a) 1/2

Given, equation is kr2 – kr – 3 = 0

If, r = 3, then, k(3)2 – k(3) – 3 = 0

⇒ 9k – 3k – 3 = 0 ⇒ 6k = 3 ⇒ k = ½

9. The ratio in which x-axis divides the join of (2, -3) and (5, 6) is:

(a) 1 : 2 (b) 3 : 4 (c) 1 : 3 (d) 1 : 5

Ans: (a) 1 : 2

Let P(x, 0) be the point on x-axis which divides the join of (2, -3) and (5, 6) in the ratio k : 1.
∴ By section formula,

$$P(x, 0) = \left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

$$\Rightarrow y = 0 \Rightarrow \frac{6k-3}{k+1} = 0 \Rightarrow 6k-3 = 0 \Rightarrow k = \frac{1}{2}$$

10. If tan θ = 1, then the value of sec θ + cosec θ is:

(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

Ans: (c) $2\sqrt{2}$

Given, tan θ = 1, we have θ = 45°

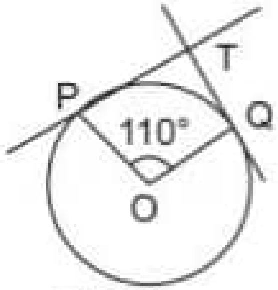
So, sec θ + cosec θ = $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

11. If the area of circle is numerically equal to twice its circumference, then the diameter of the circle is

(a) 4 units (b) 6 units (c) 8 units (d) 12 units

Ans: (c) 8 units

12. In the given figure, if TP and TQ are tangents to a circle with centre O, so that ∠POQ = 110°, then ∠PTQ is



(a) 110° (b) 90° (c) 80° (d) 70°

Ans: (d) 70°

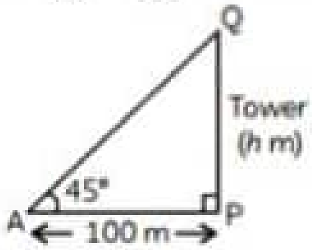
13. If the angle of elevation of the top of a tower from a point of observation at a distance of 100 m from its base is 45°, then the height of the tower is:

(a) 160 m (b) 100 m (c) 200 m (d) 150 m

Ans: (b) 100 m

Here, PQ is the tower and A is a point of observation at a distance of 100 m from PQ.

From right ΔAPQ, $\frac{PQ}{AP} = \frac{h}{100} = \tan 45^0 = 1 \Rightarrow h = 100m$



Thus, the height of tower is 100 metre.

14. If the perimeter of a circle is equal to that of a square, then the ratio of the area of circle to the area of the square is

(a) 14: 11 (b) 12: 13 (c) 11:14 (d) 13:12

Ans: (a) 14: 11

Here, it is given that 4s = 2πr $\Rightarrow s = \frac{\pi r}{2}$

$$Now, \frac{Area\ of\ Circle}{Area\ of\ Square} = \frac{\pi r^2}{s^2} = \frac{\pi r^2}{\frac{\pi^2 r^2}{4}} = \frac{4}{\pi} = \frac{4 \times 7}{22} = \frac{14}{11}$$

So, the ratio of the area of the circle to the area of square is 14 : 11.

15. For the following distribution:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

the upper limit of the median class is

(a) 18.5 (b) 20.5 (c) 25.5 (d) 17.5

Ans:

Class	Frequency	Cf
– 0.5 – 5.5	13	13
5.5 – 11.5	10	23
11.5 – 17.5	15	38
17.5 – 23.5	8	46
23.5 – 29.5	11	57

Here, $n = 57$ So, $\frac{n}{2} = 28.5$

The cumulative frequency, just greater than 28.5, is 38 which belongs to class 11.5 – 17.5.
So, the median class is 11.5 – 17.5 Its upper limit is 17.5

16. If the mean of the following distribution is 2.6, then the value of y is

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

Ans: (b) 8

Variable (x)	1	2	3	4	5	Total
Frequency (f)	4	5	y	1	2	y + 12
fx	4	10	3y	4	10	3y + 28

Here, $\sum f = y + 12$ and $\sum fx = 3y + 28$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow 2.6 = \frac{3y+28}{y+12} \Rightarrow 3y + 28 = 2.6y + 31.2$$

$$\Rightarrow 0.4y = 3.2 \Rightarrow y = 8$$

17. Two different dice are thrown together. The probability of getting the sum of the two numbers less than 7 is:

(a) 5/12 (b) 7/12 (c) 12/5 (d) 3/11

Ans: (a) 5/12

Total outcomes = 36

Number of outcomes in which sum of two numbers is less than 7 = 15

∴ Required probability = 15/36 = 5/12

18. The radii of 2 cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then, the ratio of their volumes is:

(a) 19 : 20 (b) 20 : 27 (c) 18:25 (d) 17:23

Ans: (b) 20 : 27

Let r_1 and r_2 be the two radii and h_1 and h_2 be the corresponding two heights of the two cylinders. Then

$$\frac{r_1}{r_2} = \frac{2}{3} \text{ and } \frac{h_1}{h_2} = \frac{5}{3} \text{ (Given)}$$

$\therefore \frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.
Choose the correct option

19. Assertion (A): The value of y is 3, if the distance between the points P(2, -3) and Q(10, y) is 10.

Reason (R): Distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
For y = 3
Distance PQ $= \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$ units

20. Assertion: The HCF of two numbers is 9 and their LCM is 2016. If the one number is 54, then the other number is 336.

Reason: Relation between numbers and their HCF and LCM is product of two numbers a, b = HCF (a, b) × LCM (a, b).

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

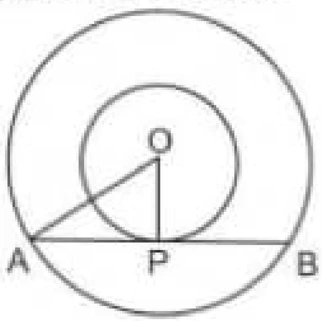
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
Let the other number be x.
$9 \times 2016 = 54 \times x$
$\Rightarrow x = 336$

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans: Let O be the centre of the concentric circle of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at P



Then AP = PB and OP⊥AB
Applying Pythagoras theorem in △OPA, we have
$OA^2 = OP^2 + AP^2 \Rightarrow 25 = 9 + AP^2$
$\Rightarrow AP^2 = 16 \Rightarrow AP = 4$ cm
∴ AB = 2AP = 8 cm

22. Evaluate: $3\cos^2 60° \sec^2 30° - 2\sin^2 30° \tan^2 60°$.
Ans: $3\cos^2 60° \sec^2 30° - 2\sin^2 30° \tan^2 60°$
$= 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2 - 2\left(\frac{1}{2}\right)^2\left(\sqrt{3}\right)^2 = \frac{3}{4}\times\frac{4}{3} - 2\times\frac{1}{4}\times 3 = 1 - \frac{3}{2} = -\frac{1}{2}$

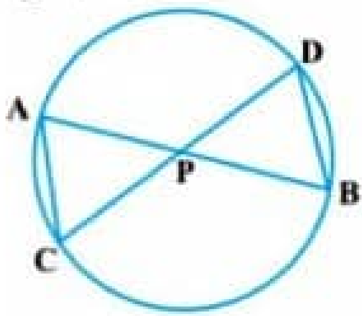
23. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
Ans: Area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.
Area of sector of angle $\theta = \frac{\theta}{360^0}\pi r^2$
Area of sector of 30° $= \frac{30^0}{360^0}\times\frac{22}{7}\times 14\times 14 = \frac{1}{12}\times\frac{22}{7}\times 14\times 14 = \frac{154}{3}$
$= 51.33$ cm²
Therefore, the area swept by the minute hand in 5 minutes is 51.33 cm²

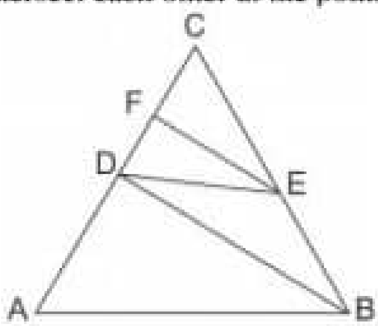
OR

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze (use π = 3.14)
Ans: Length of the rope = 5 m
Area of the field the horse can graze = Area of the sector with θ = 90° and r = 5
$= \theta/360° \times \pi r^2 = 90°/360° \times \pi r^2 = 1/4 \times \pi \times (5\text{ m})^2 = 25/4 \times 3.14\text{ m}^2 = 19.625\text{ m}^2$

24. For what value of k for which the following pair of linear equations have infinitely many solutions: 2x + 3y = 7, (k – 1)x + (k + 2)y = 3k is
Ans: For a pair of linear equations to have infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k} \Rightarrow \frac{2}{k-1} = \frac{3}{k+2}$
Now, $2k + 4 = 3k - 3 \Rightarrow k = 7$
and $9k = 7k + 14 \Rightarrow k = 7$
Hence, the value of k is 7.

25. In the below left figure, two chords AB and CD intersect each other at the point P.





Prove that (i) ΔAPC ~ ΔDPB (ii) AP. PB = CP. DP
Ans: (i) Consider Δs APC and DPB
∠APC = ∠DPB (Vertically opposite angles)

Also, ∠CAP = ∠PDB (Angles made by the same arc CB)
So, by AA similarity criteria,
ΔAPC ~ ΔDPB.
(ii) Corresponding sides of similar triangles are proportional
$\therefore \frac{AP}{PC} = \frac{DP}{PB}$
⇒ AP.PB = CP.DP

OR

If in the given above right sided figure, AB ‖ DE and BD ‖ EF, then prove that DC^2 = CF x AC

Ans: ΔABC in which DE ‖ AB and BD ‖ EF.
In ΔABC, DE ‖ AB
$\Rightarrow \frac{CD}{AC} = \frac{CE}{BC}$ ...(i) [Basic proportionality theorem]
Again in ΔCDB, EF ‖ BD
$\Rightarrow \frac{CF}{CD} = \frac{CE}{BC}$ ...(ii) [BPT]
From (i) and (ii), we get
$\frac{CD}{AC} = \frac{CF}{CD} \Rightarrow CD^2 = AC \times CF$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student 'A' takes food for 22 days, he has to pay Rs. 1380 as hostel charges; whereas a student 'B', who takes food for 28 days, pays Rs. 1680 as hostel charges. Find the fixed charges and the cost of food per day.
Ans: Let the fixed hostel charges be Rs. x and the cost of food per day be Rs. y.
According to the question, we get
x + 22y = 1380 ...(i)
and x + 28y = 1680 ...(ii)
Subtracting (i) from (ii), we get
6y = 300 ⇒ y = 300 ÷ 6 = 50
Putting y = 50 in (i), we get
x + 22(50) = 1380 ⇒ x + 1100 = 1380 ⇒ x = 280
∴ Fixed hostel charges = Rs. 280 and cost of the food per day = Rs. 50.

OR

Meena went to a bank to withdraw Rs 2,000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. How many notes of Rs. 50 and Rs. 100 she received?
Ans: Let Meena has received x no. of Rs. 50 notes and y no. of Rs. 100 notes.
So, 50x + 100y = 2000 ... (i)
x + y = 25 ... (ii)
Solving (i) and (ii), we get y = 15
Putting y = 15 in equation (ii), we get
x + 15 = 25
⇒ x = 10
Meena has received 10 pieces of Rs. 50 notes and 15 pieces of Rs. 100 notes.

27. Prove that √5 is an irrational number.

Ans: Let $\sqrt{5}$ is a rational number then we have $\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get $p^2 = 5q^2$

$\Rightarrow p^2$ is divisible by 5 $\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2 \Rightarrow q^2 = 5m^2$

$\Rightarrow q^2$ is divisible by 5 $\Rightarrow$ q is also divisible by 5

From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore, our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

28. What number should be added to the polynomial $x^2 - 5x + 4$ so that 3 is the zero of the polynomial?

Ans: Let k be the number to be added to the given polynomial.

Then the polynomial becomes $x^2 - 5x + (4 + k)$

As 3 is the zero of this polynomial, we get $(3)^2 - 5(3) + (4 + k) = 0$

$\Rightarrow (4 + k) = 15 - 9 \Rightarrow 4 + k = 6 \Rightarrow k = 2$

Thus, 2 is to be added to the given polynomial.

29. Prove that: $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^2\theta}{1-\cot\theta} = 1 + \sin\theta\cos\theta$

Ans: $LHS = \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^2\theta}{1-\cot\theta}$

$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)}{\cos\theta - \sin\theta}$

$= \cos^2\theta + \sin^2\theta + \cos\theta\sin\theta$

$= 1 + \sin\theta\cos\theta = RHS$

OR

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Ans: Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta)$

$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

30. All the black face cards are removed from a pack of 52 playing cards. The reaming cards are well shuffled and then a card is drawn at random. Find the probability of getting (i) face card (ii) red card (iii) black card.

Ans: When all the black face cards are removed.

Remaining number of cards = 52 – 6 = 46

(i) Number of face cards in the remaining deck = 6

∴ P(getting a face card) = 6/46 = 3/23

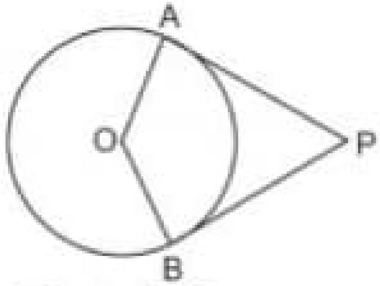
(ii) Number of red cards in the remaining deck = 26

$\therefore$ P (getting a red card) = 26/46 = 13/23

(iii) Number of black cards in the remaining deck = 20

$\therefore$ P (getting a black card) = 20/46 = 10/23

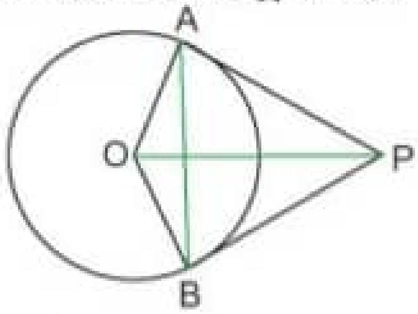
31. In the given figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.



Ans: Join OP and let it meets the circle at point Q.

Since OP = 2r (Diameter of the circle)

Consider ΔAOP in which $OA \perp AP$ and OP is the hypotenuse.



In ΔAOP, $\sin \angle OPA = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2} = \sin 30^0$

$\Rightarrow \angle OPA = 30^0$

$\Rightarrow \angle APB = 2\angle OPA = 60^0$ (Centre lies on the bisector of the angle formed between two tangents)

Now, PA = PB (Tangents from P)

$\Rightarrow \angle PAB = \angle PBA$ ($\angle$s opposite to equal sides)

Using $\angle APB$ and angle sum property of triangle, we have

$\angle PAB = \angle PBA = \angle APB$

$\Rightarrow \Delta APB$ is an equilateral triangle.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Some students planned a picnic. The total budget for food was Rs. 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by Rs. 20. How many students attended the picnic and how much did each student pay for the food?

Ans: Case I. Let number of students = x

and cost of food for each member = Rs. y

Then $x \times y = 2{,}000$ ...(i)

Case II. New number of students = $x - 5$

New cost of food for each member = Rs. $(y + 20)$

Then $(x - 5)(y + 20) = 2{,}000$

$\Rightarrow xy + 20x - 5y - 100 = 2{,}000$ ...(ii)

Solving (i) and (ii), we get $x = -20, 25$

$x = -20$ is rejected because number of students can't be negative.

So, $x = 25 \Rightarrow y = 80$

Number of students = 25

Cost of food for each student = Rs. 80.

OR

If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Ans: Let the present age of Zeba be x years.

Age before 5 years = (x – 5) years

According to given condition, $(x-5)^2 = 5x + 11$

$\Rightarrow x^2 + 25 - 10x = 5x + 11 \Rightarrow x^2 - 10x - 5x + 25 - 11 = 0$

$\Rightarrow x^2 - 15x + 14 = 0 \Rightarrow x^2 - 14x - x + 14 = 0$

$\Rightarrow x(x-14) - 1(x-14) = 0 \Rightarrow (x-1)(x-14) = 0$

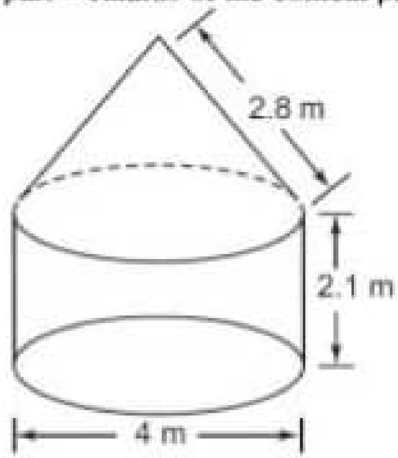
$\Rightarrow x - 1 = 0$ or $x - 14 = 0 \Rightarrow x = 1$ or $x = 14$

But present age cannot be 1 year.

Hence, Present age of Zeba is 14 years.

33. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2.

Ans: Radius (r) of the cylindrical part = Radius of the conical part = 4/2 m = 2 m



Area of the canvas used = CSA of the cylindrical part + CSA of the conical part

$= 2\pi rh + \pi rl$

$= \pi r(2h + l) = \frac{22}{7} \times 2 \times (4.2 + 2.8) = \frac{22}{7} \times 2 \times 7$

$= 44\ m^2$

Cost of the canvas = Rate per m^2 × area of canvas

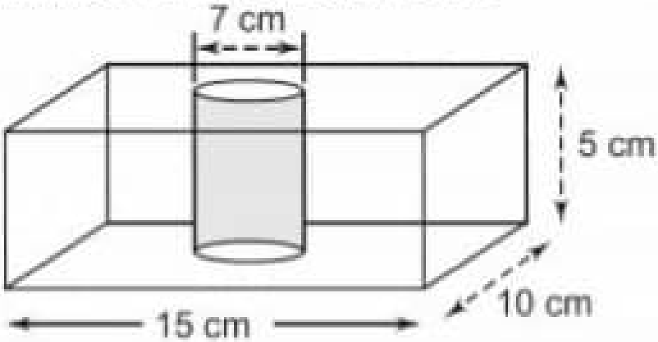
= Rs 500 × 44 = Rs. 22,000

OR

A rectangular metal block has length 15 cm, breadth 10 cm and height 5 cm. From this block, a circular hole of diameter 7 cm is drilled out. Find: (i) the volume of the remaining solid (ii) the surface area of the remaining solid.

Ans: (i) The volume of the remaining solid

= Volume of rectangular block – Volume of the circular hole



$= (15\times10\times5)-\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times5\ cm^3$

$= 750\ cm^3 - 192.5\ cm^3 = 557.5\ cm^3$

(ii) The surface area of the remaining solid

= Total surface area of the block – 2 (area of circle of the hole) + curved surface of circular hole (cylinder)

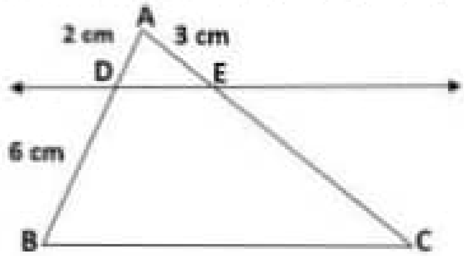
$= 2(l \times b + b \times h + h \times l) - 2(\pi r^2) + 2\pi rh$

$= 2(15\times10+10\times5+5\times15)-2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times5+2\times\frac{22}{7}\times\frac{7}{2}\times5$

$= 550\ cm^2 - 77\ cm^2 + 110\ cm^2 = 583\ cm^2$.

34. Prove that "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio."

In the figure, find EC if AD/DB = AE/EC using the above theorem.



Ans: For Theorem:

Figure, Given, To Prove and Construction - 2 marks

Proof - 2 marks

To find the value of EC = 9 cm – 1 mark

35. The distribution below gives the makes of 100 students of a class, if the median marks are 24, find the frequencies f_1 and f_2

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	4	6	10	f_1	25	f_2	18	5

Ans:

Class	Frequency	cf
0 – 5	4	4
5 – 10	6	10
10 – 15	10	20
15 – 20	f_1	$20 + f_1$
20 – 25	25	$45 + f_1$
25 – 30	f_2	$45 + f_1 + f_2$
30 – 35	18	$63 + f_1 + f_2$
35 – 40	5	$68 + f_1 + f_2$

Now, Median = 24 (Given)

So, median class = 20 – 25

For this class,

$l = 20$, $h = 5$, $N2 = 50$, $cf = 20 + f_1$, $f = 25$

We know, Median $= l + \left(\frac{\frac{n}{2}-cf}{f}\times h\right)$

$\Rightarrow 24 = 20+\frac{50-(20+f_1)}{25}\times5 \Rightarrow 4=\frac{30-f_1}{5} \Rightarrow 30-f_1=20 \Rightarrow f_1=10$

Also, sum of frequencies = 100

$\Rightarrow 68 + f_1 + f_2 = 100 \quad \Rightarrow f_1 + f_2 = 32 \Rightarrow 10 + f_2 = 32 \Rightarrow f_2 = 22$

$\therefore f_1 = 10, f_2 = 22.$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case Study – 1

Saving money is a good habit and it should be inculcated in children from the beginning. A father brought a piggy bank for his son Aditya. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

OR

If 6 times the 6th term of an A.P., is equal to 9 times the 9th term, find its 15th term.

Ans: Child's savings day wise are 5, 10, 15, 20, 25, ... to n days

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2\times1+(n-1)1]=190$

$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$

$\Rightarrow (n + 20)(n-19) = 0 \Rightarrow (n + 20)(n - 19) = 0$

$\Rightarrow n = -20$ or $n = 19$

But number of coins cannot be negative

n = 19 (rejecting n = -20)

So, number of days = 19

Total money she saved = 5 + 10 + 15 + 20 + ...

= 5 + 10 + 15 + 20 + ... upto 19 terms

$=\frac{19}{2}[2\times5+(19-1)5]=\frac{19}{2}[10+90]=\frac{19}{2}[100]=19\times50=950$

OR

Let, the first term of A.P. be 'a' and its common difference be 'd'

Given, $6a_6 = 9a_9$

$\Rightarrow 6(a + 5d) = 9(a + 8d)$

$\Rightarrow 6a + 30d = 9a + 72d$

$\Rightarrow -3a = 42d \quad \Rightarrow a = -14d \quad ...(i)$

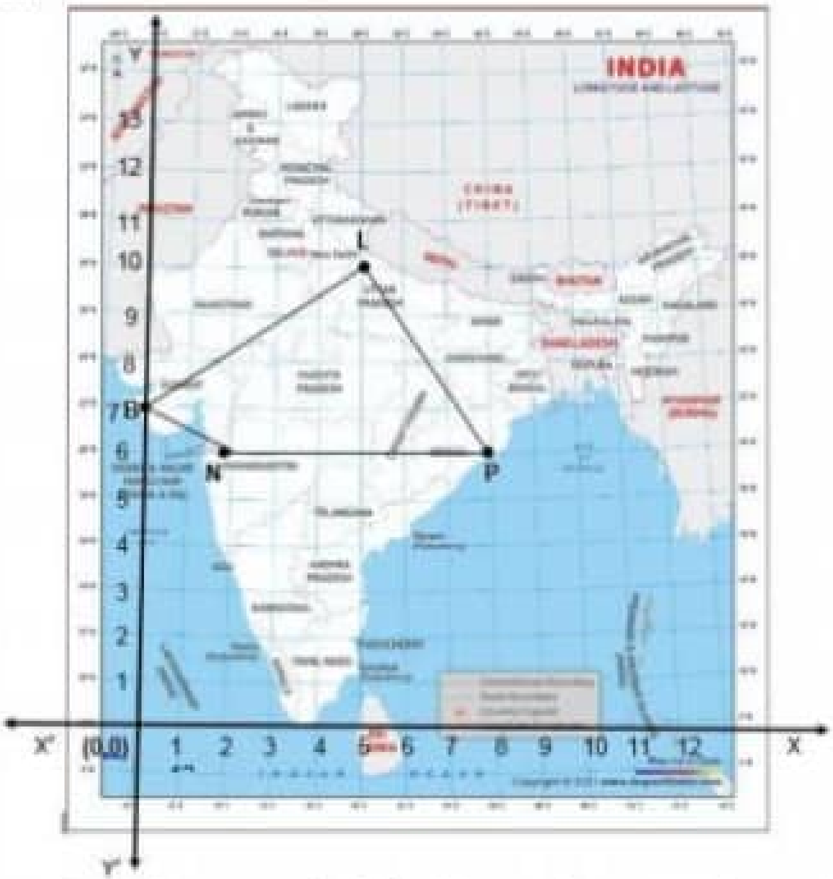
Then, 15th term i.e. $a_{15} = a + 14d = -14d + 14d$ [from (i)]

= 0

Hence, 15th term of A.P. is 0.

37. Case Study – 2

In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.



Based on the above information answer the following questions using the coordinate geometry.

(i) Find the distance between Lucknow (L) to Bhuj(B).

(ii) If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

(iii) Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

OR

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).

(i)

LB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ⇒ LB = $\sqrt{(0 - 5)^2 + (7 - 10)^2}$

LB = $\sqrt{(5)^2 + (3)^2}$ ⇒ LB = $\sqrt{25 + 9}$ LB = $\sqrt{34}$

(ii) Coordinates of Kota (K)= $\left(\frac{3\times0+2\times5}{3+2}, \frac{3\times7+2\times10}{3+2}\right) = \left(\frac{10}{5}, \frac{41}{5}\right) = \left(2, \frac{41}{5}\right)$

(iii)

L(5, 10), N(2,6), P(8,6)

LN = $\sqrt{(2 - 5)^2 + (6 - 10)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

NP = $\sqrt{(8 - 2)^2 + (6 - 6)^2} = \sqrt{(4)^2 + (0)^2} = 4$

PL = $\sqrt{(8 - 5)^2 + (6 - 10)^2} = \sqrt{(3)^2 + (4)^2}$ ⇒ LB = $\sqrt{9 + 16} = \sqrt{25} = 5$

as LN = PL ≠ NP, so Δ LNP is an isosceles triangle.

OR

Let A (0, b) be a point on the y – axis then AL = AP

$\Rightarrow \sqrt{(5-0)^2+(10-b)^2} = \sqrt{(8-0)^2+(6-b)^2}$

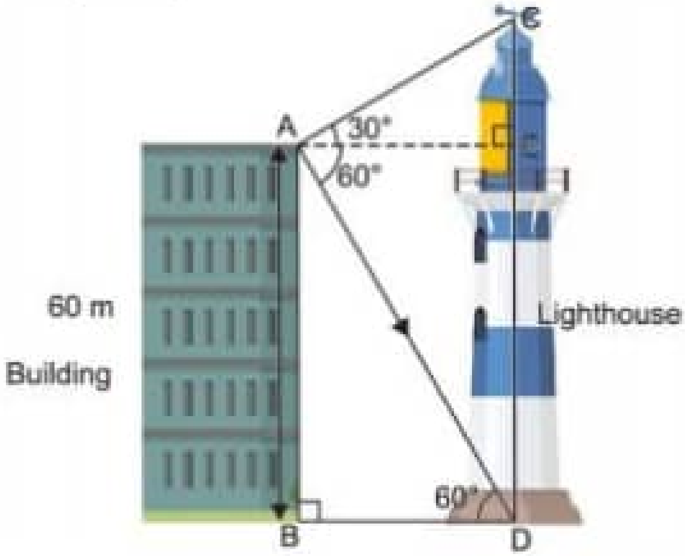
$\Rightarrow (5)^2 + (10-b)^2 = (8)^2 + (6-b)^2$

$\Rightarrow 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$

So, the coordinate on y axis is $\left(0, \frac{25}{8}\right)$

38. Case Study – 3

Ram is watching the top and bottom of a lighthouse from the top of the building. The angles of elevation and depression of the top and bottom of a lighthouse from the top of a 60 m high building are 30° and 60° respectively.



Find (i) the difference between the heights of the lighthouse and the building.

(ii) the distance between the lighthouse and the building.

OR

The ratio of the height of a light house and the length of its shadow on the ground is √3 : 1 What is the angle of elevation?

Ans: In right ΔABD,

$\tan 60^0 = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{60}{BD} \Rightarrow BD = \frac{60}{\sqrt{3}} = 20\sqrt{3}m$

∴ AE = 20√3 m (∵ BD = AE)

Now in right ΔAEC

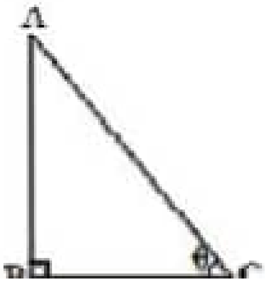
$\tan 30^0 = \frac{CE}{AE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{20\sqrt{3}} \Rightarrow CE = 20m$

(i) Difference between the heights of the lighthouse and the building = CE = 20 m

(ii) The distance between the lighthouse and the building = BD = 20√3 m.

OR

Let AB be the light house, BC be its shadow and θ be the angle of elevation of the sun at that instant



Then, in triangle ABC, we have, $\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{\sqrt{3}}{1} = \tan 60^\circ \Rightarrow \theta = 60^\circ$

Hence, angle of elevation of the sun is 60°.