

BCM SCHOOL BASANT AVENUE DUGRI ROAD LUDHIANA
CLASS XI(MATHS ASSIGNMENT)
ANSWER KEY

1	<p>$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos x}$ is equal to</p> <p>1. 0 2. -1 3. 1 4. does not exit</p> <p>ANS: 0</p>	
2	<p>$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$ is equal to</p> <p>1. 2 2. 0 3. 1 4. -1</p> <p>ANS: 1</p>	
3	<p>Differentiate $\sin^3 x \cos^3 x$ w.r.t x ANS:</p> <div style="background-color: #e0e0e0; padding: 10px;"> $\frac{dy}{dx} = \cos^3 x \frac{d}{dx}(\sin^3 x) + \sin^3 x \frac{d}{dx}(\cos^3 x)$ $\Rightarrow \frac{dy}{dx} = \cos^3 x (3 \sin^2 x \cos x) + \sin^3 x (3 \cos^2 x \cdot (-\sin x))$ $\Rightarrow \frac{dy}{dx} = 3 \sin^2 x \cdot \cos^4 x - 3 \cos^2 x \cdot \sin^4 x$ $\Rightarrow \frac{dy}{dx} = 3 \sin^2 x \cdot \cos^2 x (\cos^2 x - \sin^2 x)$ </div>	
4	<p>Find the derivative of $f(x) = \sqrt{\sin x}$, by first principle. ANS:</p>	

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(x+h)} - \sqrt{\sin x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(x+h)} - \sqrt{\sin x}}{h} \cdot \frac{\sqrt{\sin(x+h)} + \sqrt{\sin x}}{\sqrt{\sin(x+h)} + \sqrt{\sin x}} \\
&= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
f'(x) &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
&= \lim_{h \rightarrow 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
&= \lim_{h \rightarrow 0} \frac{\sin x(\cos h - 1)}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} + \frac{\cos x \sin h}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
&= \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} \frac{\sin x}{\sqrt{\sin(x+h)} + \sqrt{\sin x}} \\
&\quad + \frac{\sin h}{h} \frac{\cos x}{\sqrt{\sin(x+h)} + \sqrt{\sin x}} \\
f'(x) &= 0 \times \frac{\sin x}{\sqrt{\sin(x)} + \sqrt{\sin x}} + 1 \times \frac{\cos x}{\sqrt{\sin(x)} + \sqrt{\sin x}} \\
&= \frac{\cos x}{2\sqrt{\sin(x)}}
\end{aligned}$$

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Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$

ANS:

Given that $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$

Rationalizing the Numerator and denominator,

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{[\sqrt{1+x^3} - \sqrt{1-x^3}][\sqrt{1+x^3} + \sqrt{1-x^3}]}{x^2[\sqrt{1+x^3} + \sqrt{1-x^3}]} \\ &= \lim_{x \rightarrow 0} \frac{(1+x^3) - (1-x^3)}{x^2[\sqrt{1+x^3} + \sqrt{1-x^3}]} \\ &= \lim_{x \rightarrow 0} \frac{1+x^3 - 1 + x^3}{x^2[\sqrt{1+x^3} + \sqrt{1-x^3}]} \\ &= \lim_{x \rightarrow 0} \frac{2x^3}{x^2[\sqrt{1+x^3} + \sqrt{1-x^3}]} = \lim_{x \rightarrow 0} \frac{2x}{\sqrt{1+x^3} + \sqrt{1-x^3}} = 0 \end{aligned}$$

6 Differentiate $(3x + 5)(1 + \tan x)$ w.r.t to x

ANS:

$$\frac{dy}{dx} = \frac{d}{dx} [(3x + 5)(1 + \tan x)]$$

$$\Rightarrow \frac{dy}{dx} = (3x + 5) \frac{d}{dx} (1 + \tan x)$$

$$+ (1 + \tan x) \frac{d}{dx} (3x + 5)$$

$$\left[\because \frac{d(u \cdot v)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \right]$$

$$\Rightarrow \frac{dy}{dx} = (3x + 5) \left[\frac{d(1)}{dx} + \frac{d(\tan x)}{dx} \right] +$$

$$(1 + \tan x) \left[\frac{d(3x)}{dx} + \frac{d(5)}{dx} \right]$$

$$\left[\because \frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} \right]$$

$$\Rightarrow \frac{dy}{dx} = (3x + 5)(0 + \sec^2 x) + (1 + \tan x)(3 + 0)$$

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Evaluate $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

Ans. $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

$$= \lim_{h \rightarrow 0} \frac{(a^2 + 2ah + h^2) \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a^2 \sin(a+h) + 2ah \sin(a+h) + h^2 \sin(a+h) - a^2 \sin a}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a^2 [\sin(a+h) - \sin a] + 2ah \sin(a+h) + h^2 \sin(a+h)}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{a^2 2 \cos \left[\frac{2a+h}{2} \right] \sin \frac{h}{2}}{2 \frac{h}{2}} + \lim_{h \rightarrow 0} 2a \sin(a+h) + \lim_{h \rightarrow 0} h \sin(a+h) \\ &= a^2 \cos \left[\frac{2a+0}{2} \right] \times 1 + 2a \sin[a+0] + 0 \times \sin a \\ &= a^2 \cos a + 2a \sin a \end{aligned}$$