	BCM SCHOOL BASANT AVENUE DUGRI ROAD LUDHIANA ASSIGNMENT CLASS XIISC	
1	**Which of the following is the principal value branch of $\cos^{-1}x$?** **A)$[0,\pi]$ B)$(0,\pi)$ C)$\{0,\pi\}$ D)$(0,\pi]$**	1
2	Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos\left(\tan^{-1}\sqrt{3}\right)$ A) $\frac{29}{26}$ B) $\frac{31}{26}$ C) $\frac{37}{29}$ D) $\frac{37}{26}$	1
3	Find the principal value of **$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$** We have, $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ $= \tan^{-1}(\sqrt{3}) - \{\pi - \cot^{-1}(\sqrt{3})\}$ [$\because \cot^{-1}(-x) = \pi - \cot^{-1}x; x \in R$] $= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$ $= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi$ $= \frac{\pi}{2} - \pi = -\frac{\pi}{2}$ [$\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}; x \in R$] Which is the required principal value.	2
4	Write the value of **$\tan^{-1}[2\sin(2\cos^{-1}(\sqrt{3}/2))]$** We have, $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $= \tan^{-1}\left[2\sin\left\{2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right\}\right]$ $\left[\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$ $= \tan^{-1}\left[2\sin\left\{2\times\frac{\pi}{6}\right\}\right]$ $[\because \cos^{-1}(\cos\theta) = \theta; \forall\theta\in[0,\pi]]$ $= \tan^{-1}\left(2\sin\frac{\pi}{3}\right) = \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right)$ $\left[\because \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right]$ $= \tan^{-1}(\sqrt{3}) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$ $\left[\because \tan\frac{\pi}{3} = \sqrt{3} \text{ and } \tan^{-1}(\tan\theta) = \theta, \forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$	2
5	Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$	2

Let $\cos^{-1}\dfrac{4}{5} = x$, then

$$\Rightarrow \quad \cos x = \frac{4}{5}$$

$$\Rightarrow \quad \sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$\Rightarrow \quad \sin x = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\Rightarrow \quad \tan x = \frac{3}{4}$$

$$\Rightarrow \quad x = \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

Now,

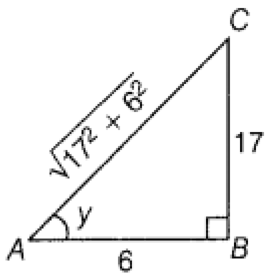
$$\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) = \sin\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$$

$$= \sin\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1\right]$$

$$= \sin\left(\tan^{-1}\left(\frac{\frac{9+8}{12}}{\frac{12-6}{12}}\right)\right) = \sin\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$$

Now, let $\tan^{-1}\left(\dfrac{17}{6}\right) = y$



$$\Rightarrow \quad \tan y = \frac{17}{6}$$

$$\Rightarrow \quad \sin y = \frac{17}{\sqrt{325}} = \frac{17}{5\sqrt{13}}$$

$$\Rightarrow \quad y = \sin^{-1}\left(\frac{17}{5\sqrt{13}}\right)$$

$$= \tan^{-1}\left(\frac{17}{6}\right)$$

Hence,

$$\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) = \sin\left(\sin^{-1}\frac{17}{5\sqrt{13}}\right) = \frac{17}{5\sqrt{13}}$$

$$[\because \sin(\sin^{-1}x) = x,\ x \in [-1, 1]]$$

6	Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}$ To Prove $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$ $\text{LHS}=\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$ Put $\cos^{-1}\frac{a}{b}=\theta \Rightarrow \cos\theta=\frac{a}{b}$ $\text{LHS}=\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ $=\dfrac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}+\dfrac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$ $=\dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\dfrac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$ $=\dfrac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)\left(1+\tan\frac{\theta}{2}\right)}$ $=2\left(\dfrac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)=\dfrac{2}{\cos\theta}=\dfrac{2}{a/b}=\dfrac{2b}{a}$ $\left[\because \cos\theta=\dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right]$ $=\text{RHS}$ **Hence proved.**	3
7	Solve the following equation for x. **$\cos(\tan^{-1}x)=\sin(\cot^{-1}3/4)$**	4

We have, $\cos(\tan^{-1} x) = \sin(\cot^{-1} \frac{3}{4})$ .... (i)

Let $\tan^{-1} x = \theta$ and $\cot^{-1} \frac{3}{4} = \Phi$ $\forall \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and $\Phi \in (0, \pi)$

$\Rightarrow \tan \theta = x$ and $\cot \Phi = \frac{3}{4}$

$\Rightarrow \sec \theta = \sqrt{1+\tan^2\theta}$ and $\operatorname{cosec} \Phi = \sqrt{1+\cot^2\phi}$

[taking positive square root as $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\Phi \in (\theta, \pi)$]

$$\Rightarrow \quad \sec\theta = \sqrt{1+x^2}$$

$$\text{and } \operatorname{cosec}\phi = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{1}{\cos\theta} = \sqrt{1+x^2} \text{ and } \frac{1}{\sin\phi} = \frac{5}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{1+x^2}} \text{ and } \sin\phi = \frac{4}{5}$$

$$\Rightarrow \quad \theta = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \text{ and } \phi = \sin^{-1}\frac{4}{5}$$

$$\Rightarrow \quad \tan^{-1} x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \text{ and } \cot^{-1}\frac{3}{4} = \sin^{-1}\frac{4}{5}$$

On substituting these values in Eq. (i), we get

$$\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \quad [\because \cos(\cos^{-1} x) = x;\ \forall x \in [-1,1]$$

$$\text{and } \sin(\sin^{-1} x) = x;\ \forall x \in [-1,1]]$$

On squaring both sides, we get

$$16(x^2+1) = 25 \Rightarrow 16x^2 = 9 \Rightarrow x^2 = \frac{9}{16}$$

$\Rightarrow x = \pm\frac{3}{4}$ [taking square root both sides]

But $x = \frac{-3}{4}$ does not satisfy the given equation. Hence, the required solution is $x = \frac{3}{4}$.

Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

Answer:

To prove, $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

$$\text{LHS} = \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) \qquad \ldots(i)$$

Let $\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = \theta \qquad \ldots(ii)$

Then, $\sin^{-1}\left(\frac{3}{4}\right) = 2\theta \Rightarrow \sin 2\theta = \frac{3}{4}$

Also, $2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now, $\sin 2\theta = \frac{3}{4}$

$\Rightarrow \quad \frac{2\tan\theta}{1+\tan^2\theta} = \frac{3}{4} \qquad \left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow \quad 8\tan\theta = 3 + 3\tan^2\theta$

$\Rightarrow \quad 3\tan^2\theta - 8\tan\theta + 3 = 0$

Now, by quadratic formula

$$\tan\theta = \frac{-(-8) \pm \sqrt{(-8)^2 - 4\times 3\times 3}}{2\times 3}$$

$\Rightarrow \quad \tan\theta = \frac{8 \pm \sqrt{64-36}}{6} = \frac{8 \pm \sqrt{28}}{6}$

$\Rightarrow \quad \tan\theta = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$

As, $-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$\Rightarrow \quad -1 \le \tan\theta \le 1$

$\therefore \quad \tan\theta = \frac{4-\sqrt{7}}{3} \qquad \left[\because \frac{4+\sqrt{7}}{3} > 1\right]$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4-\sqrt{7}}{3}\right)$$

$[\because \tan\theta = x \Rightarrow \theta = \tan^{-1} x]$

$$\Rightarrow \quad \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{4-\sqrt{7}}{3}\right)$$

[from Eq. (ii)]

On taking tan both sides, we get

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \tan\left\{\tan^{-1}\left(\frac{4-\sqrt{7}}{3}\right)\right\}$$

$$\therefore \quad \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3} = \text{RHS}$$