SAMPLE QUESTION PAPER-4
(Explanations)

1. (a) Total number of outcomes $= 6$

total number of lying between 2 and 6 $= 3$

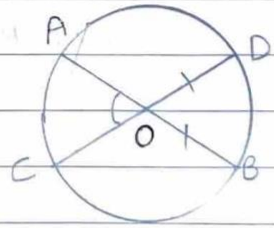
therefore,

Probability $= \frac{3}{6} = \frac{1}{2}$

2. (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

3. (d) Length of $OB = OD$ and $OC = OA$

So, ΔOBD and ΔOAC are isosceles.



Now, in ΔAOC and ΔBOD

$OA = OB$ (radius of circle)

$OC = OD$ (" " ")

$\angle AOC = \angle BOD$ (vertical opp. $\angle$'s)

$\therefore \Delta AOC \cong \Delta BOD$ (SAS)

4. (c) one root is 2

So put $x = 2$

$4 + 2b + 12 = 0$

$2b = -16 \Rightarrow b = -8$

Now, roots of equation $x^2 + bx + q = 0$ are equal, $\Rightarrow$ $\underline{q = 16}$..

5. (d) We need to convert the angle to radius:

$$\theta = \frac{120}{180}\pi = \frac{2}{3}\pi$$

The formula for the length of an arc is:

$$S = r\theta$$

Substituting the given values

$$\pi = r \cdot \frac{2}{3}\pi \Rightarrow r = \frac{3}{2} \text{ cm.}$$

6. (a) According to question we have,

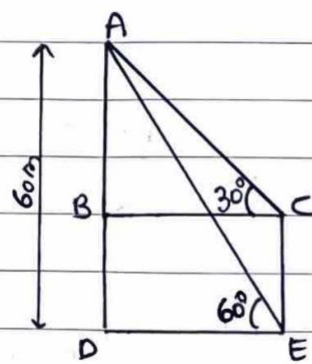
$$2(x+10) = 2x + 3x + 2$$

$$\Rightarrow \quad 2x + 20 = 5x + 2$$

$$\Rightarrow \quad 3x = 18$$

$$\Rightarrow \quad x = 6$$

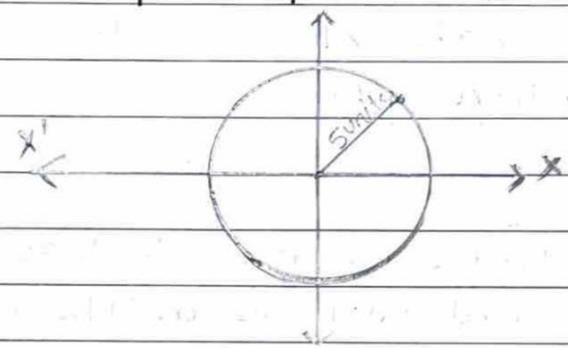
7. (a) $\tan 60^\circ = \frac{60}{DE} \Rightarrow DE = \frac{60}{\sqrt{3}} = 20\sqrt{3}$



$\tan 30^\circ = \frac{AB}{20\sqrt{3}} \Rightarrow AB = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20$

Now, as $AD = AB + BD \Rightarrow 60 = 20 + BD \Rightarrow BD = 40$ m.

8. (d) There are infinite points on the circle in 3^{rd} quadrant.



9. (a) $ax^2 + 2x + a = 0$

Here $a = a$, $b = 2$, $c = a$

We know that $D = b^2 - 4ac$

Putting these values,

$= (2)^2 - 4 \times a \times a$

$= 4 - 4a^2$

$\therefore D \geq 0$

Since roots are equal $D = 0$

$\therefore 4 - 4a^2 = 0 \Rightarrow 4a^2 = 4 \Rightarrow a^2 = \frac{4}{4} \Rightarrow a = \pm 1$

10. (c) $HCF = 2$, $LCM = 36$

$\because$ product of both $= HCF \times LCM$

$\therefore a \times 18 = 2 \times 36$

$a = 4.$

11. (d) This is a rational number. It can be expressed as $\frac{0}{1}$, which is the ratio of two integers expressed in the form $\frac{p}{q}$ where $q \neq 0$.

12. (a) The quadratic equation $4x^2+3x+2=0$ can be written in the standard form as $ax^2+bx+c=0$, where $a=4$, $b=3$ and $c=2$.

The sum of the coefficients of x^2, x and the constant term is given by $a+b+c$.

Substituting the values of a, b and c we get;

$$a+b+c = 4+3+2 = 9.$$

13. (c) The given equation is : $6x^2-9x-220$.

here $a=6$, $b=-9$, $c=-220$.

Discriminant $D = b^2-4ac$

$$= (-9)^2 - 4\times 6\times(-220)$$

$$= 81 + 5280$$

$$= 5361$$

$\therefore$ $D = 5361 > 0$, therefore, the roots are real and unequal.

14. (b) Given equations are

$$x+2y+7=0$$
$$2x+ky+14=0$$

For coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{2}{k} = \frac{7}{14}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{k} \Rightarrow k=4.$$

15. (a) $\because$ volume of a cone $= \frac{1}{3}\pi r_2^2 h_2$

volume of cylinder $= \pi r_2^2 h_2$

According to question,

$$\frac{r_1}{r_2} = \frac{2}{3} \quad \& \quad \frac{h_1}{h_2} = \frac{4}{5}$$

So, ratio $= \frac{\text{Volume of cone}}{\text{Volume of cylinder}}$

$$= \frac{1/3\,\pi\,(2/3)^2 \times 4}{\pi \qquad\qquad 5} \Rightarrow \frac{1}{3} \times \frac{4}{9} \times \frac{4}{5} \Rightarrow 16:135.$$

16. (a) The sum of the roots of the equation is given by

$$a+b+c = \frac{k}{1}$$

$$abc = \frac{n}{1} = n$$

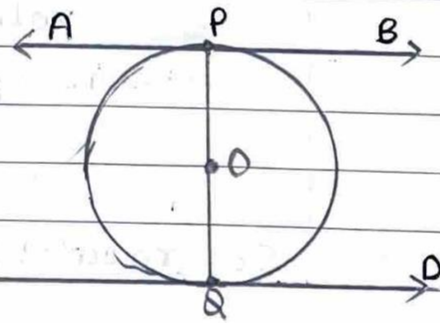
Now, $\frac{1}{ca} + \frac{1}{ab} + \frac{1}{bc} = \frac{b+c+a}{abc}$

$$\Rightarrow \frac{k}{n}$$

17 (d) $PQ = 2 \times PO$ {Distance b/w two parallel tangents is equal to the diameter}

$$= 2 \times 5 \text{ cm}$$

$$= 10 \text{ cm}$$



18. (d) From the distance formula, we have

$$= \sqrt{(8-0)^2+(-6-0)^2} = \sqrt{64+36} = \sqrt{100}$$

$$= 10 \text{ units}.$$

19. (C) Assertion (A) is true but Reason (R) is false.

Since the product of zeroes is $\frac{10}{1} = 10$

For $ax^3 + bx^2 + cx + d$, product of roots $= -\frac{d}{a}$

20. (C) Assertion (A) is true but Reason (R) is false.

An integer is a rational number, so the assertion is true. Whereas root of any integer can't be said as irrational.

21. b)

$m\sin A + n\cos A = p$ —— ①

$(m\sin A + n\cos A)^2 = p^2$ (squaring both sides).

$m^2\sin^2 A + n^2\cos^2 A + 2mn\sin A\cdot\cos A = p^2$ —— ②

$(m\cos A - n\sin A)^2 = q^2$

$m^2\cos^2 A + n^2\sin^2 A - 2mn\cos A\cdot\sin A = q^2$ —— ④

Adding ② and ④

$m^2(\sin^2 A + \cos^2 A) + n^2(\sin^2 A + \cos^2 A) + 0 = p^2 + q^2$

$m^2 + n^2 = p^2 + q^2$ Hence proved.

22. $x^2+5x+6=0 \Rightarrow x^2+2x+3x+6=0$

$$x(x+2)+3(x+3)=0$$

$$(x+2)(x+3)=0$$

$$x=-2,-3$$

$$\therefore \alpha=-2,\ \beta=-3$$

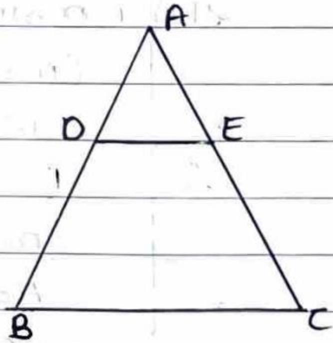
$$\because \alpha+\beta=\frac{-b}{a} \Rightarrow -2-3=\frac{-5}{1} \Rightarrow -5=-5$$

$$\because \alpha\beta=\frac{c}{a} \Rightarrow (-2)(-3)=\frac{6}{1} \Rightarrow 6=6.$$

23. Given:

In ΔABC, D is midpoint of AB and DE is parallel to BC.

$\therefore$ AD = DB.



To proove: AE = EC

Proof: Since, DE ∥ BC

$\therefore$ By Basic proportionality theorem,

$$\frac{AD}{DB}=\frac{AE}{EC}$$

$\therefore$ AD = DB, Therefore, $\frac{AE}{EC}=1 \Rightarrow AE=EC.$

24. (a) Let P, Q, R and S are points where circle touches the sides AB, BC, CD and DA respectively.

Therefore,

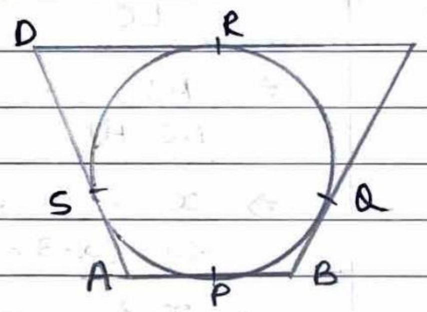
$$AB + CD = BC + AD$$

$$6 + 8 = 9 + AD$$

$$14 = 9 + AD$$

$$AD = 14 - 9$$

$$AD = 5$$



Length of AD is 5 cm.

25. Given that $\operatorname{cosec}(\theta) = \sec(60^\circ)$

and we know that $\sec(\theta) = \frac{1}{\cos(\theta)}$ & $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

we can write this as,

$$\frac{1}{\sin\theta} = \frac{1}{\cos(60^\circ)}$$

Since, $\cos(60^\circ) = \frac{1}{2}$ we have, $\frac{1}{\sin\theta} = 2 \Rightarrow \sin\theta = \frac{1}{2}$

Since θ is a positive acute angle, and in the first quadrant, $2\sin^2(\theta) - 1$ at $\theta = 30^\circ$

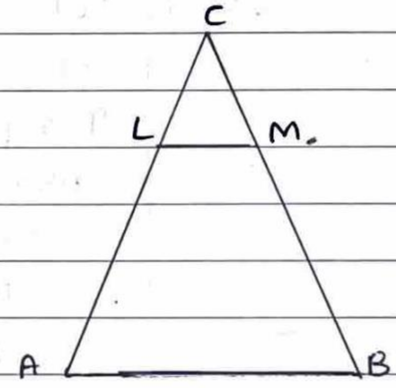
$$\Rightarrow 2\sin^2(30^\circ) - 1 = 1 - 1 = 0$$

So, the value of $2\sin^2(\theta) - 1$ is 0.

26. (b) In ΔABC we have

$LM \parallel AB$

$\therefore \frac{AL}{LC} = \frac{BM}{MC}$ [By thales Theorem]



$\Rightarrow \frac{AL}{AC-AL} = \frac{BM}{BC-BM}$

$\Rightarrow \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$

$\Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5}$

$\Rightarrow (x-3)(x+5) = (x-2)(x+3)$

$\Rightarrow x^2+2x-15 = x^2+x-6$

$\Rightarrow x = 9.$

27. (b) $\tan A = n \tan B \Rightarrow \tan B = \frac{1}{n} \tan A \Rightarrow \cot B = \frac{n}{\tan A}$

$\sin A = m \sin B \Rightarrow \sin B = \frac{1}{m} \sin A \Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$

substituting the values of cot B and cosec B.

In $\operatorname{cosec}^2 B - \cot^2 B = 1$

We get,

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow m^2 - 1 = \cos^2 A \, (n^2 - 1)$$

$$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2 A .$$

20. Let $p(x) = x^2+x-12$

and $q(x) = 2x^2-kx-9$

Let's suppose, $h(x) = (x-k)$ be HCF of $p(x)$ and $q(x)$

Now, $p(x) = x^2+x-12$

$= x^2+4x-3x-12$

$= x(x+4)-3(x+4)$

$= (x+4)(x-3)$

So, either $k=-4$ or $k=3$ . . . . . (i)

Since, $(x-k)$ is HCF.

$\therefore$ $q(x) = (x-k)\,g(x)$ for some factor $g(x)$ of $q(x)$.

Take $x=k$

$\Rightarrow$ $2(k)^2 - k\times k - 9 = 0$

$\Rightarrow$ $2k^2 - k^2 - 9 = 0$

$\Rightarrow$ $k^2 = 9 \Rightarrow k = \pm 3$

from (i) we get

$k = 3$.

29. Given, $r = 3.5$ cm, $h = 10$ cm

T.S.A of the article = 2 × CSA of the hemispherical part + CSA of the cylindrical part.

$= 2 \times 2\pi r^2 + 2\pi r h$

$= 2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times 3.5 (2 \times 3.5 + 10)$

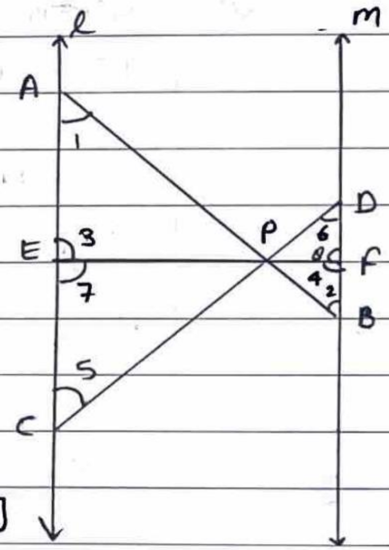
$= \frac{22}{3.5} \times 3.5 (7+10)$

$= 22 \times 17$

$= 374 \text{ cm}^2$

Thus, the total surface area of the article is 374 cm^2.

30. To proove: $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$



Proof: In ΔAEP and ΔBFP,

$l \parallel m$ (Given)

$\angle 1 = \angle 2$ & $\angle 3 = \angle 4$ [Alternate interior angle]

$\therefore$ $\Delta AEP \sim \Delta BFP$, [By AA similarity]

$$\frac{AE}{BF} = \frac{AP}{BP} = \frac{EP}{FP} \text{ ------(i)}$$

In ΔCEP & ΔDEP, $l \parallel m$ (Given)

$\angle 7 = \angle 8$ & $\angle 5 = \angle 6$ [Alternate interior angles]

$\therefore$ $\Delta CEP \sim \Delta DFP$ [By AA similarity] $\Rightarrow \frac{CE}{DF} = \frac{CP}{DP} = \frac{EP}{FP}$ ---(ii)

In ΔACP and ΔBDP $l \parallel m$ (Given)

$\angle 1 = \angle 2$ & $\angle 5 = \angle 6$ [Alternate interior angles]

$\therefore$ $\Delta ACP \sim \Delta BDP$ [By AA similarity]

$$\frac{AC}{BD} = \frac{AP}{BP} = \frac{CP}{DP} \text{ -----(iii)}$$

From (i), (ii) and (iii)

$$\frac{AP}{PB} = \frac{AC}{BD} = \frac{CP}{DP} = \frac{CE}{DF} = \frac{EP}{FP} = \frac{AE}{BF} \Rightarrow \frac{AC}{BD} = \frac{AE}{BF} = \frac{CE}{DF}.$$

31. Given,

$\angle OAB = \angle OBA = 30°$ [OA = OB = radius]

We know that,

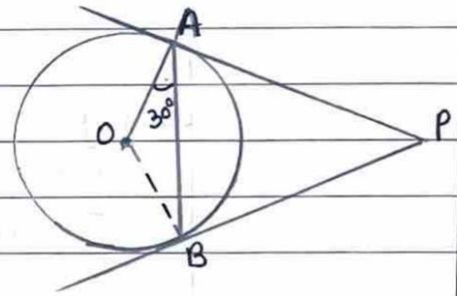
$\angle OAP = \angle OBP = 90°$ [Tangents]

$\Rightarrow \angle OAB + \angle BAP = 90°$

$\Rightarrow 30 + \angle BAP = 90°$

$\Rightarrow \angle BAP = 60°$

Similarly,



$\angle OBA + \angle ABP = 90°$

$\Rightarrow 30° + \angle ABP = 90°$

$\Rightarrow \angle ABP = 60°$

In ΔABP

$\angle BAP + \angle ABP + \angle APB = 180°$

$\Rightarrow 60° + 60° + \angle APB = 180°$

$\Rightarrow \angle APB = 180° - 120°$

$= 60°$

Hence $\angle APB = 60°$

32. (b) Length of minute hand = radius of the clock.

$\therefore$ Radius (r) of the circle = 14 cm (Given)

Angle swept by minute hand in 60 minutes = 360°

So, the angle swept by the minute hand in 5 minute $= \frac{360^\circ}{60^\circ} \times 5 = 30^\circ$

We know,

Area of sector $= \frac{\theta}{360} \times \pi r^2$

Now area of the sector making an angle of 30°

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{1}{12} \times 22 \times 2 \times 14$

$= \frac{154}{3}$ cm^2.

Hence, the required area swept by the minute hand in 5 minutes is $\frac{154}{3}$ cm^2.

33.

Marks	f	c.f.
20-30	p	p
30-40	15	15+p
40-50	25	40+p
50-60	20	60+p
60-70	q	60+p+q
70-80	8	68+p+q
80-90	10	78+p+q

We are given median = 50

Thus sum of all the frequencies = 90

$\Rightarrow p+15+25+20+q+8+10 = 78+p+q = 90$

$\Rightarrow p+q = 12$

Also,

$$\text{median} = l + \frac{(x/2 - c.f)}{f} \times h$$

$$50 = 40 + \frac{(45-(15+p))}{25} \times 10 \Rightarrow 25 = 30 - p$$

$-p = -5$ or $p = 5$, here $p+q = 12 \Rightarrow q = 12-p \Rightarrow q = 7$

$\therefore p = 5,\ q = 7$

mode occurs in (40-50) as 25 is highest frequency, mode $= 40 + \left[\frac{25-15}{2\times25-15-20}\right]$

$= 46.67.$

34. (a) Let the father's age be x and son's age be y.

By I condition:

$$3+3y = x$$

By II condition:

$$x+3 = 2(y+3)+10$$

So final 2 equations are

$$x-3y = 3 \quad \ldots\ldots \text{(i)}$$

$$x-2y = 13 \quad \ldots\ldots \text{(ii)}$$

(i) − (ii)

$$-y = -10 \Rightarrow y = 10$$

Substituting $y=10$ in (1)

$$x-(3\times 10) = 3$$

$$x-30 = 3$$

$$x = 3+30$$

$$\Rightarrow x = 33$$

Solving these equations we get,

$$y = 10 \ \& \ x = 33$$

So, the father's present age is 33 yrs. and that of the son is 10 yrs.

35. Jacks and Queens of black colour are removed ⇒ 2 each is removed = 4 cards.

kings and Aces of red colour are removed.

⇒ 2 each card of (king & Ace) removed = 4 cards.

(a) A black king:

Total number of black king = 2.

and total cards are $52-8 = 44$.

$\therefore$ $P(\text{black king}) = \frac{2}{44} = \frac{1}{22}$.

(b) A red colour card:

No. of red colour card = $26-4 = 22$.

$P(\text{red card}) = \frac{22}{44} = \frac{1}{2}$

(c) No. of black coloured jacks in a deck = 0

$P(\text{black jack}) = \frac{0}{44} = 0$

(d) No. of face cards in the deck = $12-6=6$

$P(\text{face card}) = \frac{6}{44} = \frac{3}{22}$.

36.

(i) Money saved on I day = Rs. 27.5

$\because$ Sehaj increases his savings by a fixed amount of Rs. 2.5.

$\therefore$ His saving from an AP with $a = 27.5$ and $d = 2.5$

$\therefore$ Money saved on 10^{th} day.

$$a_{10} = a + 9d = 27.5 + 9(2.5)$$
$$= 27.5 + 22.5$$
$$= Rs.\ 50$$

(ii) Money saved on I day = Rs. 27.5

money saved on 25^{th} day

where $a = 27.5$ and $d = 2.5$

$$a_{25} = a + 24d$$
$$= 27.5 + 24(2.5) = Rs.\ 87.5$$

(iii) Total amount saved by Sehaj in 30 days.

$$S_{30} = \frac{30}{2}\left[2 \times 27.5 + (30-1) \times 2.5\right]$$

$$= 15\left[55 + 29(2.5)\right] = 1912.5.$$

37\. (i) Let D be (a,b) then

mid-point of AC = midpoint of BD

$$\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$$

$\therefore$ $4+a = 7 \Rightarrow a = 3$

$3+b = 8 \Rightarrow b = 5$

central midfielder is at (3,5)

(ii) $GH = \sqrt{(-3-3)^2+(5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$GK = \sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$

$HK = \sqrt{(3-0)^2+(1-3)^2} = \sqrt{9+4} = \sqrt{13}$

$GK + HK = GH$

$\Rightarrow$ G, H and K Lie on a same straight Line.

(iii) A, B and E lie on the same straight Line and B is equidistant from A and E.

$\Rightarrow$ B is the mid-point of AE

$$\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2,-3), \quad 1+a=4 \;;\; a=3$$
$$4+b = -6 \;;\; b = -10$$

$\therefore$ F is $(3,-10)$

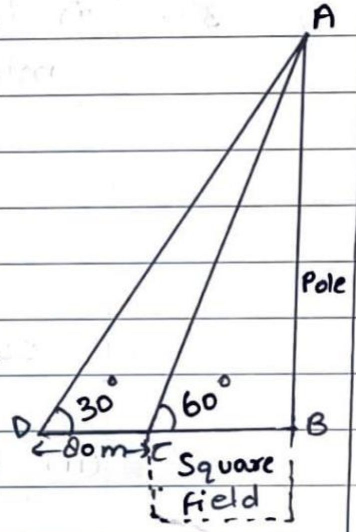
38. (i) In ΔABC

$$\tan 60^\circ = \frac{x}{l}$$

$$\sqrt{3} = \frac{x}{l} \Rightarrow x = \sqrt{3}\,l \text{ - - - - - (i)}$$

Now $l = 40$ metres

from eq. (i) $x = \sqrt{3}\,l = 40\sqrt{3} = 69.28$ metres.



(ii) OR, Distance from farmer at position D and top of pole is AD. Hence in ΔABC;

$$\cos 30^\circ = \frac{DB}{AD} \Rightarrow AD = \frac{DB}{\cos 30^\circ} \Rightarrow AD = \frac{120}{\frac{\sqrt{3}}{2}} = \frac{240}{\sqrt{3}}$$

$AD = 138.56$ m.

(iii) In ΔABC, $\tan 60^\circ = \frac{x}{l} \Rightarrow x = \sqrt{3}\,l$ - - - - (i)

In ΔABD, $\tan 30^\circ = \frac{x}{80+l} \Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,l}{80+l}$ (From eq.(i))

$$\Rightarrow 80 + l = 3l \Rightarrow 2l = 80 \Rightarrow l = 40$$

Thus length of the field is 40 metres.