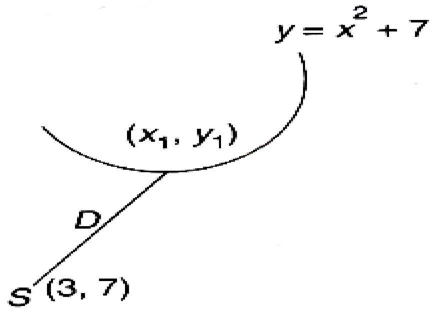
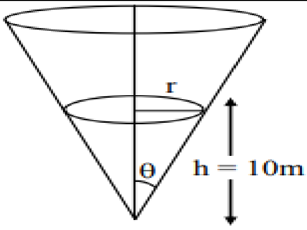
	BCM SCHOOL BASANT AVENUE DUGRI LUDHIANA ASSIGNMENT APPLICATION OF DERIVATIVE	
1	The maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$. (a) 25 (b) 43 (c) 62 (d) 49	d
2	The sides of an equilateral triangle are increasing at the rate of 2 cm/s. The rate at which the area increases, when the side is 10 cm, is (a) √3 cm2/s (b) 10 cm2/s (c) 10√3 cm2/s (d) $\frac{10}{\sqrt{3}}$ cm2/s	c
3	The radius of a cylinder is increasing at the rate of 3 m/s and its height is decreasing at the rate of 4 m/s. The rate of change of volume when the radius is 4 m and height is 6 m, is (a) 80π cm^3/s (b) 144π cm^3/s (c) 80 cm^3/s (d) 64 cm^3/s	c
4	the total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.	126
5	Find the intervals in which the function given by; $f(x) = 3/10\, x^4 - 4/5x^3 - 3x^2 + 36/5x + 11$ is (i) strictly increasing. (ii) strictly decreasing.	decreasing in (-∞, -2) and (1, 3) increasing in (-2,1) (3,∞)
6	An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.	$\sqrt{5}$units

	 $\because (x_1, y_1)$ lie on curve $y = x^2 + 7$ $\therefore \quad y_1 = x_1^2 + 7$ or $\quad D^2 = (x_1 - 3)^2 + (x_1^2 + 7 - 7)^2$ or $\quad D^2 = x_1^2 - 6x_1 + 9 + x_1^4$ Differentiate D^2 w.r.t. x_1, we get $\frac{d(D^2)}{dx_1} = 2x_1 - 6 + 4x_1^3$ $= 4x_1^3 + 2x_1 - 6$ Now, $\frac{d(D^2)}{dx_1} = 0$ or $\quad 4x_1^3 + 2x_1 - 6 = 0$ or $\quad 4x_1^2(x_1 - 1) + 4x_1(x_1 - 1) + 6(x_1 - 1) = 0$ or $\quad (x_1 - 1)(4x_1^2 + 4x_1 + 6) = 0$ or $\quad x_1 - 1 = 0$ or $4x_1^2 + 4x_1 + 6 = 0$ or $\quad x_1 = 1$ $\because$ [$4x_1^2 + 4x_1 + 6$ have no real roots] Again $\frac{d^2(D^2)}{dx_1^2} = 12x_1^2 + 2$ $\left.\frac{d^2(D^2)}{dx_1^2}\right]_{x_1=3} = +\text{ve}$ Hence, for $x_1 = 1$, D^2 is minimum, *i.e.*, D is minimum. Also, for $x_1 = 1, y_1 = 1^2 + 7 = 8$ $\therefore$ Minimum distance, $D = \sqrt{(1-3)^2 + (8-7)^2}$ $= \sqrt{5}$ unit	
7	A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}$ (0.5). Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.	

	 given $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ $\Rightarrow \tan\theta = \frac{1}{2} = \frac{r}{h}$ $\Rightarrow r = \frac{h}{2}$ $v = \frac{1}{3}\pi r^2 h$ $v = \frac{1}{3}\pi \frac{h^3}{4}$ $\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$ $S = \frac{\pi}{4}(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$	
8	A telephone company in a town has 500 subscribers on its list and collects fixed charges of ₹ 300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of ₹ 1/- one subscriber will discontinue the service. Find what increase will bring maximum profit? ANS Let us consider that the company increases the annual subscription by ₹ x. So, x is the number of subscribers who discontinue the services. ∴ Total revenue, $R(x) = (500 - x)(300 + x)$ $= 150000 + 500x - 300x - x^2$ $= -x^2 + 200x + 150000$ Differentiating both sides w.r.t. x, We get $R'(x) = -2x + 200$ For local maxima and local minima, $R'(x) = 0$ $-2x + 200 = 0$ $\Rightarrow x = 100$ $R''(x) = -2 < 0$ Maxima So, $R(x)$ is maximum at $x = 100$	

9 **Case-Study :** Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(i) If the length and the breadth of the rectangular field be $2x$ and $2y$ respectively, then find the area function in terms of x.

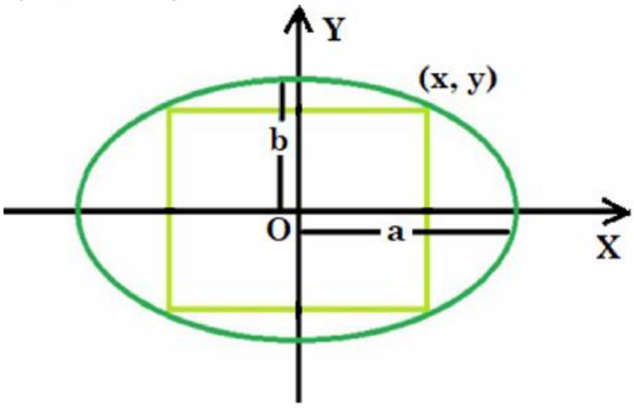
(ii) Find the critical point of the function.

(iii) Use First Derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

OR

Use Second Derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area. *(2022-23)*

(i) Let $(x\,,\,y)=\left(x\,,\,\frac{b}{a}\sqrt{a^2-x^2}\right)$ be the upper right vertex of the rectangle.



The area function, $A=2x\times 2\frac{b}{a}\sqrt{a^2-x^2}$

$\Rightarrow A=\frac{4b}{a}x\sqrt{a^2-x^2}\,,\; x\in(0,\,a).$

(ii) $\frac{dA}{dx}=\frac{4b}{a}\left[x\times\frac{-x}{\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}\right]$

$\Rightarrow\frac{dA}{dx}=\frac{4b}{a}\times\frac{a^2-2x^2}{\sqrt{a^2-x^2}}=-\frac{4b}{a}\times\frac{(\sqrt{2}x+a)(\sqrt{2}x-a)}{\sqrt{a^2-x^2}}$

For $\frac{dA}{dx}=0 \quad\Rightarrow x=\frac{a}{\sqrt{2}}$

So, $x=\frac{a}{\sqrt{2}}$ is the critical point.

(iii) For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx}>0$ and for the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx}<0$.

Hence, by the first derivative test, there is a local maximum at the critical point $x=\frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x=\frac{a}{\sqrt{2}}$.

$\therefore$ For maximum area of the soccer field, its length should be $a\sqrt{2}$ units and its width should be $b\sqrt{2}$ units.

OR

(iii) $A=2x\times 2\frac{b}{a}\sqrt{a^2-x^2}\,,\; x\in(0\,,\,a)$.

Squaring both sides, we get $Z=A^2=\frac{16b^2}{a^2}x^2(a^2-x^2)=\frac{16b^2}{a^2}(x^2a^2-x^4),\; x\in(0\,,\,a)$.

($\because$ A is maximum when Z is maximum.)

Now $\frac{dZ}{dx}=\frac{16b^2}{a^2}(2xa^2-4x^3)=\frac{32b^2}{a^2}x(a+\sqrt{2}x)(a-\sqrt{2}x)$

For $\frac{dZ}{dx}=0 \quad\Rightarrow x=\frac{a}{\sqrt{2}}$

Also, $\frac{d^2Z}{dx^2}=\frac{32b^2}{a^2}(a^2-6x^2)$

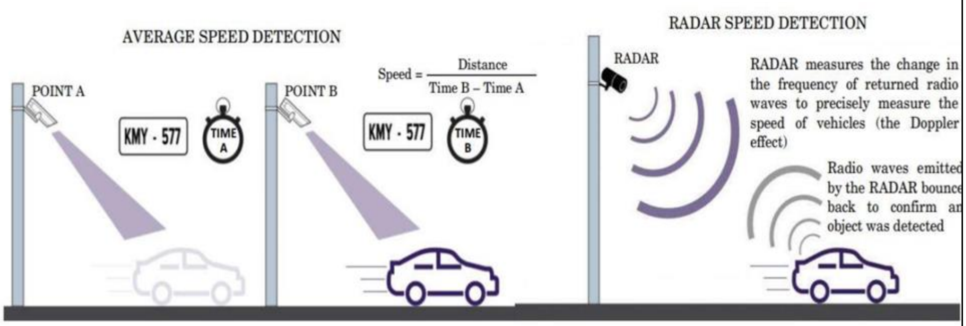
As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}}=\frac{32b^2}{a^2}(a^2-3a^2)=-64b^2<0$.

So, by the second derivative test, there is a local maximum value of Z at the critical point $x=\frac{a}{\sqrt{2}}$.

Since there is only one critical point therefore, Z is maximum at $x = \frac{a}{\sqrt{2}}$.

Hence, A is maximum at $x = \frac{a}{\sqrt{2}}$.

10 The traffic police have installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions:

(i) Express θ in terms of height of the camera installed on the pole and x.

(ii) Find $\frac{d\theta}{dx}$

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

(iii)(b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is 3/101 rad/s, then find the speed of the car.

(i) Express θ: $\theta = \tan^{-1}\left(\frac{5}{x}\right)$.

(ii) $\frac{d\theta}{dx} = -\frac{5}{x^2+25}$.

(iii) $\frac{d\theta}{dt} = -\frac{4}{101}$ rad/s.

(iii)(b) Speed =15 m/s