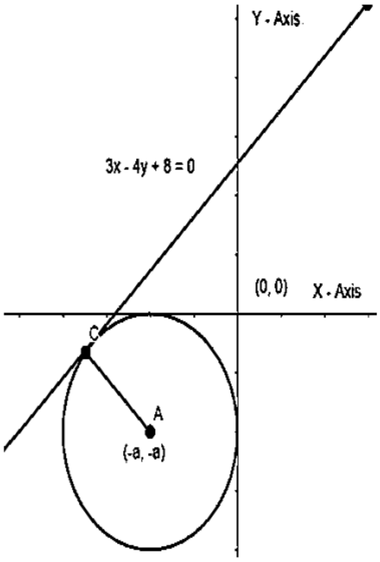
	BCM SCHOOL LUDHIANA ANSWER KEY
1	$\left(\frac{a}{2}, \frac{b}{2}\right)$
2	$x^2 + y^2 - 2x - 4y + 1 = 0$
3	Since both lines are parallel & tangent to the circle. Then distance between these two lines must be the diameter of the circle. **Lines:** $3x - 4y + 4 = 0$ $3x - 4y - 3.5 = 0$ **(Equating the co-efficient of the equation)** Distance (between 2 || lines) = $\left(\frac{\|c-p\|}{\sqrt{a^2+b^2}}\right)$ for the two given equations, $ax + by + c = 0$ & $ax + by + p = 0$ Distance = $\left(\frac{\|4-(-3.5)\|}{\sqrt{3^2+(-4)^2}}\right) = \left(\frac{\|7.5\|}{\sqrt{9+16}}\right) = \frac{7.5}{5} = 1.5$ units $\because$Radius $= \frac{\text{Diameter}}{2} = \frac{1.5}{2}$ = 0.75 units. **Hence, the radius of the circle is 0.75 units.**

4

The line which touches the circle is $3x-4y+8=0$, which is a tangent to the circle.

$\therefore$ The perpendicular distance = a units (radius of the circle)



$$a = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$a = \frac{|3(-a) - 4(-a) + 8|}{\sqrt{3^2 + 4^2}}$$

$$a = \frac{|-3a + 4a + 8|}{\sqrt{9+16}}$$

$$a = \frac{|a+8|}{\sqrt{25}}$$

$$a = \frac{a+8}{5}$$

$$5a = a + 8$$

$$a = 2$$

Co-ordinates of the centre of the circle = (-2,-2)

Since, the equation of a circle having centre (h,k), having radius as "r" units, is

$$(x-h)^2 + (y-k)^2 = r^2$$

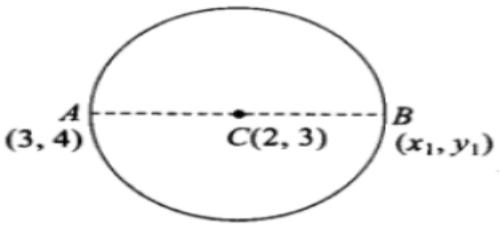
$$(x-(-2))^2 + (y-(-2))^2 = 2^2$$

$$(x+2)^2 + (y+2)^2 = 4$$

$$x^2 + 4x + 4 + y^2 + 4y + 4 - 4 = 0$$

$$x^2 + y^2 + 4x + 4y + 4 = 0$$

5



Given equation of the circle is

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

$\therefore$ $2g = -4$ and $2f = -6$

So, the centre of the circle is $C(-g, -f) \equiv C(2, 3)$

$A(3, 4)$ is one end of the diameter.
Let the other end of the diameter be $B(x_1, y_1)$.
Here, mid point of AB is C.

$\therefore$ $2 = \frac{3 + x_1}{2}$ and $3 = \frac{4 + y_1}{2}$

$\Rightarrow$ $x_1 = 1$ and $y_1 = 2$

So, the coordinates of other end of the diameter are (1,2)

6

Solving the given equations, $3x + y = 14$ ....1 $2x + 5y = 18$ 2
point of intersection is (4, 2)
the required expression is $x^2 - 2x + y^2 + 4y - 20 = 0$

7

Given equation of the circle is:

$$x^2 + y^2 - 6x + 12y + 15 = 0$$

or $$(x-3)^2 + (y+6)^2 = (\sqrt{30})^2$$

Hence, centre is (3, –6) and radius is $\sqrt{30}$

Since the required circle is concentric with above circle, centre of the required circle is (3, –6).

Let its radius be r.

Now it is given that,

Area of the required circle

= 2 × Area of the given circle

$$\Rightarrow \quad \pi r^2 = 2 \times \pi(\sqrt{30})^2$$

$$\Rightarrow \quad r^2 = 60 \quad \Rightarrow \quad r = \sqrt{60}$$

So, equation of the required circle is:

$$(x-3)^2 + (y+6)^2 = 60$$

$$\Rightarrow \quad x^2 + y^2 - 6x + 12y - 15 = 0$$

8

Let the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Length of major axis = 2a

Length of minor axis = 2b

And the length of latus rectum = $\frac{2b^2}{a}$

We have $\frac{2b^2}{a} = \frac{2b}{2}$

$\Rightarrow b = \frac{a}{2}$

Now $b^2 = a^2(1 - e^2)$, where e is the eccentricity

$\Rightarrow b^2 = 4b^2(1 - e^2)$

$\Rightarrow 1 = 4(1 - e^2)$

$\Rightarrow 1 - e^2 = \frac{1}{4}$

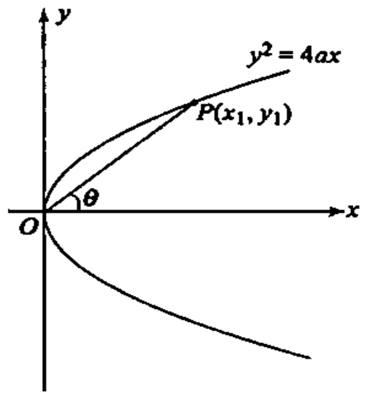
$\Rightarrow e^2 = 1 - \frac{1}{4}$

$\Rightarrow e^2 = \frac{3}{4}$

$\therefore e = \pm\frac{\sqrt{3}}{2}$

So, $e = \frac{\sqrt{3}}{2}$ ......[∵ e is not (–)]

Hence, the required value of eccentricity is $\frac{\sqrt{3}}{2}$.

9	Given equation of the parabola is $y^2 = 4ax$. Let the point on the parabola be $P(x_1, y_1)$.  From the figure, slope of $OP = \tan\theta = \frac{y_1}{x_1}$ (i) Also, $y_1^2 = 4ax_1$ (ii) Now, $OP = \sqrt{x_1^2 + y_1^2} = \sqrt{x_1^2 + \tan^2\theta x_1^2}$ $= \sqrt{x_1^2 \sec^2\theta} = x_1 \sec\theta$ From (i) and (ii), we have $\tan^2\theta x_1^2 = 4ax_1 \Rightarrow x_1 = \frac{4a}{\tan^2\theta}$ $OP = \frac{4a\sec\theta}{\tan^2\theta} = \frac{4a\cos\theta}{\sin^2\theta}$
10	We have Area of circle = 154 $\pi r^2 = 154$ $\Rightarrow r^2 = 49$ The intersection of two lines will give us the centre of the circle. Solving $2x - 3y = 5$ and $3x - 4y = 7$ we get $x = 1$ and $y = -1$ Now, the equation of the circle is given by $(x-h)^2 + (y-k)^2 = r^2$ $\Rightarrow (x-1)^2 + (y+1)^2 = 49$ $\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$
11	**The equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is the semi-major axis** **Accordingly, $2a = 10 \Rightarrow a = 5$** **Distance between the foci ($2c$) = 8** **$\Rightarrow c = 4$** **On using the relation $c = \sqrt{a^2 - b^2}$, we obtain**

	$4=\sqrt{25-b^2}$ $\Rightarrow 16=25-b^2$ $\Rightarrow b^2=25-16=9$ $\Rightarrow b=3$ **Thus, the equation of the path traced by the man is** $\frac{x^2}{25}+\frac{y^2}{9}=1$
12	**PA+PB=10** $\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$ $\Rightarrow \sqrt{(x+4)^2+y^2+z^2}=10-\sqrt{(x-4)^2+y^2+z^2}$ ...(i) $\Rightarrow (x+4)^2+y^2+z^2=100+(x-4)^2+y^2+z^2-20\sqrt{(x-4)^2+y^2+z^2}$ [On squaring both sides of (i)] $16x=100-20\sqrt{(x-4)^2+y^2+z^2}$ $\Rightarrow 5\sqrt{(x-4)^2+y^2+z^2}=(25-4x)$ $\Rightarrow 25[(x-4)^2+y^2+z^2]=625+16x^2-200x$ $\Rightarrow 9x^2+25y^2+25z^2-225=0$ Hence, the required equation of the curve is $9x^2+25y^2+25z^2-225=0.$