

BCM SCHOOL, BASANT AVENUE, DUGRI ROAD, LUDHIANA
Class XII (Continuity and differentiability)

1.	The derivative of $\tan^{-1} \left(\frac{1+x}{1-x} \right)$ with respect to x is (a) $\frac{1}{1+x^2}$ (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{2}{1-x^2}$	1
2.	The derivative of 2^x w.r.t 3^x is (a) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (b) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$ (c) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (d) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$	1
3.	$\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} =$ (a) 1 (b) -1 (c) 0 (d) None of these	1
4.	If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$.	2
5.	Find the relationship between a and b so that the function f defined by: $f(x) = \begin{cases} ax + 1, & x \leq 3 \\ bx + 3, & x > 3 \end{cases}$ is continuous at $x = 3$.	2
6.	Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$ w.r.t. $\tan^{-1} x$.	3
7.	If $x^{16}y^9 = (x^2 + y)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$	3
8.	If $\log(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x} \right)$, show that $\frac{dy}{dx} = - \frac{(x+y)}{(x-y)}$.	3
9.	If $y = \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \sec^{-1} \left(\frac{1+x^2}{1-x^2} \right)$, prove that $\frac{dy}{dx} = \frac{4}{1+x^2}$.	5
10.	If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function. When a given relation expresses y as an implicit function of x and we want to differentiate every term w.r.t x, remembering that a term in y is first differentiated w.r.t. y and then multiplied by $\frac{dy}{dx}$. Based on the above information, answer the following questions: (i) Find $\frac{dy}{dx}$, if $x^3 + x^2y + xy^2 + y^3 = 5$ (ii) If $x^y = e^{x-y}$, find $\frac{dy}{dx}$. (iii) Find $\frac{dy}{dx}$, if $e^{\sin y} = xy$. OR Find $\frac{dy}{dx}$, if $y = (\sin x)^{\sin x \sin x \dots \infty}$	4