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| | **BCM SCHOOL BASANT AVENUE DUGRI ROAD LUDHIANA**<br>**ASSIGNMENT ANSWER KEY**<br>**(RELATION AND FUNCTIONS)**<br>**CLASS XII SC** |
| **1** | **A** |
| **2** | **C** |
| **3** | **Assertion is true, reason is false** |
| **4** | **Given, f(x) = \|x\| + x and g(x) = \|x\| − x, ∀ x ∈ R.**<br>$\Rightarrow \quad f(x) = \begin{cases} x + x, & x \geq 0 \\ -x + x, & x < 0 \end{cases}$<br>and $\quad g(x) = \begin{cases} x - x, & x \geq 0 \\ -x - x, & x < 0 \end{cases}$<br>$\Rightarrow f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ and $g(x) = \begin{cases} 0, & x \geq 0 \\ -2x, & x < 0 \end{cases}$<br>**Thus, for x ≥ 0,gof (x) = g(f(x)) = g(2x) = 0**<br>**and for x < 0, gof(x) = g(f(x)) = g(0) = 0 ⇒ gof(x) = 0, ∀ x ∈ R Similarly, for x > 0, fog (x) = f(g(x)) = f(0) = 0**<br>**and for x < 0, fog (x) = f(g(x)) = f(-2x)**<br>**= 2(-2x) = -4x**<br>$\Rightarrow fog\ (x) = \begin{cases} 0, & x \geq 0 \\ -4x, & x < 0 \end{cases}$ **We have, gof(x) = 0, ∀ x ∈ R.**<br>and $fog(x) = \begin{cases} 0, & x>0 \\ -4x, & x<0 \end{cases}$<br>**Clearly, fg(-3) = -4(-3) = 12,**<br>**fog(5) = 0 and gof(-2) = 0** |
| **5** | R is Reflexive if (a, b) R (a, b) for (a, b) in N × N<br>Let (a, b) R (a, b) ⇒ a + b = b + a which is true since addition is commutative on N. ⇒<br>R is reflexive.<br>R is Symmetric if (a, b) R (c, d) ⇒ (c, d) R (a, b) for (a, b), (c, d) in N × N<br>Let (a, b) R (c, d)<br>⇒ a + d = b + c<br>⇒ b + c = a + d<br>⇒ c + b = d + a [since addition is commutative on N]<br>⇒ (c, d) R (a, b)<br>⇒ R is symmetric.<br>R is Transitive if (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f) |

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| | for (a, b), (c, d),(e, f) in N × N<br>Let (a, b) R (c, d) and (c, d) R (e, f)<br>⇒ a + d = b + c and c + f = d + e<br>⇒ (a + d) – (d + e) = (b + c) – (c + f)<br>⇒ a - e = b - f<br>⇒ a + f = b + e ⇒ (a, b) R (e, f)<br>⇒ R is transitive. |
| **6** | gof (x) = $g\left(\frac{3x+4}{5x+7}\right)=\frac{7\left(\frac{3x+4}{5x+7}\right)+4}{5\left(\frac{3x+4}{5x+7}\right)-3}=x$<br><br>$fog(x)=f\left(\frac{3x+4}{5x-3}\right)=\frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7}=x$<br><br>*Thus* gof(x) = x, for all x ∈ B<br>fog (x) = x, for all x ∈ A<br><br>Which implies that gof = IB<br><br>And Fog = IA |
| **7** | and $f(x_1) = f(x_2)$<br>$\frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$<br>$x_1(1 + x_2^2) = x_2(1 + x_1^2)$<br>$x_1 + x_1.x_2^2 - x_2 - x_2x_1^2 = 0$<br>$(x_1 - x_2).(x_1.x_2 - 1) = 0$<br>Taking $x_1 = 4,\ x_2 = \frac{1}{4} \in R.$<br>$\therefore$ $f$ is not one-one.<br>**(ii) $f$ is onto :** Let $y \in R$ (co-domain)<br>$f(x) = y$<br>$\Rightarrow \frac{x}{1+x^2} = y \Rightarrow y.(1 + x^2) = x$<br>$\Rightarrow yx^2 + y - x = 0$<br>$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$<br>since, $x \in R$, $\therefore$ $1 - 4y^2 \geq 0$<br>$\Rightarrow (1 + 2y)(1 - 2y) \geq 0$<br>$\therefore -\frac{1}{2} \leq y < \frac{1}{2}$<br>So Range $(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$<br>Range $(f) \neq R$ (Co-domain)<br>$\therefore$ $f$ is not onto. |

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| 8 | **One-One :** Let $x_1, x_2 \in R$<br>Such that $f(x_1) = f(x_2)$<br>or $\frac{x}{1+\|x_1\|} = \frac{x_2}{1+\|x_2\|}$<br>**Case (i) :** If $x_1, x_2 < 0$ then<br>$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$<br>or $x_1 - x_1x_2 = x_2 - x_1x_2$<br>or $x_1 = x_2$<br>**Case (ii) :** If $x_1, x_2 > 0$ then<br>$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$<br>$x_1 + x_1x_2 = x_2 + x_1x_2$<br>or $x_1 = x_2$<br>**Case (iii) :** If $x_1 > 0, x_2 < 0$ (Similarly for $x_1 < 0, x_2 > 0$)<br>or $\frac{x_1}{1+\|x_1\|} \neq \frac{x_2}{1-\|x_2\|}$<br>$x_1 \neq x_2$<br>or $f(x_1) \neq f(x_2)$<br>$\therefore$ From (i), (ii) & (iii) $f$ is a one-one function.<br>**Onto :** Let any $y \in \{x \in R: -1 < x < 1\}$<br>$(\because -1 < y < 1)$<br>Such that $y = f(x)$<br>or $y = \frac{x}{1+\|x\|}$ 1<br>or $y = \frac{x}{1 \pm x}$ or $x = \frac{y}{1 \pm y}$<br>As $y \neq -1,\ y \neq 1$<br>$\therefore$ $x = \frac{y}{1 \pm y} \in R$ 1<br>$\therefore$ $f$ is an onto function.<br>$f^{-1}(x) = \begin{cases} \frac{x}{1+x}, \text{ if } x < 0 \\ \frac{x}{1-x}, \text{ if } x \geq 0 \end{cases}$ |
| 9 | **fing gof and fog** |
| 10 | **(i)** $6^2$<br>**(ii)** **(d) None of these three**<br>**(iii)** **Reflexive and Transitive**<br>**(iv)** $2^{12}$ |